UNIVERSITÀ DEGLI STUDI
DI PERUGIA

ISSN 2385-2275

Working papers of the
Department of Economics
University of Perugia (IT)

Bayesian SUR Heteroskedastic Model for Demand Elasticity Analysis in the Italian Wholesale Electricity Market

Maria Chiara D'Errico

Working paper No. 20

December 2015

Bayesian SUR Heteroskedastic Model for Demand Elasticity Analysis in the Italian Wholesale Electricity Market.

Maria Chiara D'Errico

University of Perugia

Manuscript

2015

Abstract

The Italian electricity sector undertook a deregulation process starting in the 2004 that has led to overcome the system of vertically integrated monopoly. This process led to the institution of Power Exchange (IPEX). The transition had not been simple since the definition of a proper market structure preserving competition is not an immediate task. In this context, the information provided by demand elasticity have to be exploited since the elasticity is strictly linked with the market power measured on the supply side. The work want to investigate what is the extent of buyer's elasticity and if buyers can change their consumption profiles within the day given the rational expectation of change in price. The research use a Bayesian approach applying a heteroskedastic SUR regression Model .

JEL Classification: D43

Keywords: Demand Elasticity; Bayesian Estimation; Electricity Markets

1 The Italian Power Exchange

Electricity industry is a leading industrial sector since it is a fundamental input for the production processes in any industrialised country. Its strategic importance for economic development and its social and environmental impact imposes an effective regulation. For this reason it is not surprising that the electric sector was regulated by public commissions and the tariffs were kept fixed over long periods of time.

In the last decades liberalization process started in most of the developed countries, the ownership in the electricity sector became private and industry has been split up into the different functions.

The liberalization of the electricity sector has led to overcome the system of vertically integrated monopoly. Generation and retail functions have become open to competition.

Transition from state-owned monopolies to competitive markets was not always smooth and concerns had been raising in many countries; market structure affects in fact competition and for this reason the design of deregulated electricity markets offer economists a changeling opportunity. They have been attempting to design well functioning markets that gives players the correct incentives to improve production efficiency and limit market power. In the recent years many economists have focused on the effects that market design may have on equilibrium prices market power of supplier. The market structure affects in fact the consumer reactivity to change in price, that is the elasticity.

As in other international experiences, the creation of the Italian Electricity market (IPEX) responded to two specific requirements:

• promoting competition in electricity generation, sale and purchase, under criteria of neutrality, transparency and objectivity, through the creation of a market place;

• ensuring the economic management of an adequate availability of ancillary services.

The organization and the management of the Italian electricity market has been entrusted GME. Unlike other European markets, Italian Power Exchange is not a purely financial market aimed only to the definition of prices and quantities, but it is a physical market where injection and withdrawal profiles are scheduled and really delivered.

The Electricity Market is articulated in the Spot Electricity Market (MPE), Forward Electricity Market and the Financial Derivatives Market (IDEX). The Spot Electricity Market is divided into three submarkets:

The **Day-Ahead Market (MGP)**, which is the venue for the trading of electricity supply offers and demand bids for each hour of the next day.

All electricity operators may participate in the MGP. GME accepts Offers and Bids by the merit order, taking into account the current transmission constraints. Accepted supply offers are remunerated at the Zonal Clearing Price, while accepted demand bids are remunerated at the National Single Price (PUN). The accepted Offers/Bids determine the preliminary Injection and Withdrawal Schedules of each Offer Point for the next day.

The **Intra-Day Market (MI)**, which has replaced the existing Adjustment Market, it is venue for the trading of electricity supply offers and demand bids which modify the Injection and Withdrawal Schedules resulting from the Day-Ahead Market. GME accepts the Offers and Bids submitted into the MI by merit order, taking into account the Transmission Limits remaining after the Day-Ahead Market. Accepted Offers and Bids are remunerated at the Zonal Clearing Price and they Bids modify the preliminary schedules determining the revised injection and withdrawal schedules for the next day.

The **Ancillary Services Market (MSD)** is the venue for the trading of supply offers and demand bids in respect of ancillary services. This market is essentially used to acquire resources for relieving intra-zonal congestions, procuring Reserve Capacity and balancing the injections and withdrawals in the real time. Participation in the MSD is restricted to units that are authorised to supply ancillary services and to their dispatching users. Participation in the MSD is mandatory.

We focus on the Day-Ahead Market (MGP) where hourly blocks of electricity are traded for the next day are negotiated. In this market both the injection and withdrawal programs for the next day are defined in order to reach the equilibrium prices and quantities. The MGP is organized according to an implicit double auction model and the most of the transactions takes place in this market. The session opens at 8 a.m. on the ninth day before the delivery-day and closes at 9.15 a.m. on the day before the delivery is executed.

During the session, market participants submit offers to buy or sell that indicate the amount of energy and the maximum price (or the lowest price) at which they are willing to buy (or sell). In particular:

- The offers to buy (BID) represent the willingness to purchase an amount of energy that does not exceed that specified in the offer at a price no higher than that reported in the same offer.

- The offers to sell (OFF) express instead the willingness to sell an amount of energy not greater than that specified in the offer and at a price not lower than that indicated in the same offer. In the supply side operators can relate offers only to the injection points. If the offer is accepted, the producer undertakes to enter in the network, in a given period, the amount of electricity specified in the offer.

Each offer, to sale and purchase, must be consistent with the physical constraints of the corresponding unit point. The Day-Ahead Market is a zonal market, reflecting the structure which the national transmission grid is divided in. Each zone is characterized by an insufficient interconnection capacity and when a congestion occurs the selling price is zonal differentiated: selling price is lower in the upstream area of congestion and higher in the downstream ones. In depth, when the market session closes, the GME starts the process for the resolution of the market. For each hour of the next day, the algorithm accepts all the bids and offers in order to maximize the value of trading, within the limits of maximum transit between zones.

The process of acceptance can be summarized as follows:

All offers to sell are sorted according an ascending price order forming aggregate supply curve,while bids are ordered by descending price order drawing the aggregate demand curve.

The intersection between the two curves derives the total quantity traded, the equilibrium price, the accepted BID and OFF.

If electricity flows resulting from the programs do not violate any transition constraints, the equilibrium price is unique for all the zones. The accepted offers to sale are those whose sale prices are not higher than the equilibrium price, while the accepted bids are those whose purchase prices is not lower than the equilibrium price.

If at least one transmission constraint is violated, sale price are zonal differentiated and the algorithm starts the so called "Market Splitting Mechanism". It splits in fact the market into two zones, one for the export, which includes all zones upstream of the bond, and one for the import, which includes all areas downstream of the bond, repeating in each of the two areas the process described above: i.e. it derives in each zone the corresponding aggregate supply and demand curve. The outcome are two equilibrium zonal price zone (p_{z_1} and p_{z_2}). In particular, p_{z_i} is greater in the area of import and is smaller in the area of export. If, within each zone, the resulting equilibrium quantities violate further transition constraints, the splitting market process goes on within the zones in order to obtain an outcome consistent with the grid constraints.

With regard to the purchase price of electricity, GME has implemented an algorithm that, given congestion and differentiated zonal sale prices, apply just a single national purchase price (PUN), that is the average of the zonal sale prices weighted with the zonal consumptions. The PUN applies only to withdrawal points belonging to national geographical areas.

The mechanism of market splitting is an "implicit auction" for the non-discriminatory allocation of the transit rights.

2 Theoretical Background

Since the early 1970s, when energy caught the attention of policy makers in the aftermath of the first oil crisis, research on energy demand has vastly increased in order to overcome the limited understanding of the nature of energy demand and demand response due to the presence of external shocks encountered at that time.

Elasticity, in the energy demand analysis framework, is a feature that has received particular attention in the studies of consumer preference and willingness to pay, as in the institutional studies guiding policy decisions as taxation and welfare. Moreover, the consumer reactivity to changes in price can express market efficiency. Then, in strategic economic sectors, this measure can be seen as a tool leading the National Regulators in the market structure definition processes. Previous empirical studies used data referring the supply side of electricity market, given the assumption of oligopolistic market structure, they estimate demand elasticity using residual demand function. Bigerna et al. (2014a)'s work has been the first Italian study of electricity demand elasticity using data referring the demand side.

Following this approach, this work estimated demand elasticity using the same type of data. The main participants in the Italian Electricity wholesale Market are industrial consumers using power as an input in the production function to produce goods and services, while residential consumers have a domestic use of electricity. Industrial agents choose the amount of electricity input which minimizes their cost function given the technological constraint, while residential customers are part of optimizing utility function process given the budget constraint. For this reason our econometric approach will lie inside the neoclassical framework and will be grounded on rational optimizing behaviour theory.

Although data available refers only market prices and demand, the duality approach gives us a theoretical justification, allowing to legitimately switch from agent's preference (optimization theory) to market demand (The Marshallian demand) in which quantities are functions of prices and total expenditure. We assume all the agent taking part in the MGP rationally behave minimizing a cost function, (production cost function for industrial buyers and expenditure function for the residential ones).

Recalling the tradition introduced by Brendt and Wood (Berndt and

Wood, 1975) the cost function assumed is the trans-log cost function, that is the the second order approximation of an agent's cost function. Its general form can be written as follow:

$$\begin{aligned}\ln C = \ & \alpha_0 + \sum \alpha_i \ln p_i + \tfrac{1}{2} \sum_i \sum_j \gamma_{ij} \ln p_i \ln p_j + \alpha_Q \ln Q \\ & + \tfrac{1}{2}\gamma_{QQ}(\ln Q)^2 + \sum_i \gamma_{Q_i} \ln Q \ln p_i \end{aligned} \tag{1}$$

where C is the total cost, i and j are the inputs (for industrial consumers) or the other good for residential customers, p_i is the factor or good prices, Q is the objective variable (the objective variable to be maximized: it can be the output quantity or the consumer's utility)

This cost function must satisfies certain properties:

- Homogeneous of degree 1 in prices;
- Satisfying all the conditions guaranteeing a well-behaved production (or utility) function
- Homothetic (separable function of the objective variable and prices).

Minimization problem is usually solved using Lagrangian techniques, leading to the first order condition:

$$\frac{\partial C(Q,p)}{\partial p_i} = h_i(Q,p) = q_i \text{ for all } i \tag{2}$$

Under the given assumptions, solving the problem yields to a demand functions expressed in terms of prices and the objective variable: $q_i = h_i(Q,p)$. These functions are the Hicksian demands or the compensated demand equations because they consider the objective variable Q as a constant parameter. For empirical works the optimization model need to be linked to economical model in which quantities are a function of prices and total expenditure. The duality approach is the theoretical framework allowing to shift from the production possibility sets (and the system of preferences) to the market demand function.

Given the convexity of production possibility sets (or convex preferences for end consumers), the Roy Identity allows to derive Marshallian demand from the Hicksian demand substituting the objective variable Q in the Hicksian demand with the profit function (or the indirect utility function).

First we derive the Minimum Expenditure function and we put it into profit function or the indirect utility function $V(m,p)$, substituting m with $C(Q,p)$ evaluated at the optimum level. This lead to the trivial identity:

$$V(C(Q,p),p) = Q(m,p) \tag{3}$$

where $Q(m,p)$ is the utility/profit function of the maximization problem, p is the price vector and m is the budget constraint. This says that the indirect profit/utility function $V(C(Q,p),p)$, that minimizes the cost for achieving a given level of utility given a set of prices, is equal to that utility function u (of the maximization problem) evaluated at those prices . Taking the derivative of both sides of this equation with respect to the price of a single input/good p_i (with the Q's level held constant) gives:

$$\frac{V(C(Q,p),p)}{\partial Q} \cdot \frac{\partial C(Q,p)}{\partial p_i} + \frac{V(C(Q,p),p)}{\partial p_i} = 0 \tag{4}$$

Rearranging what we obtain is:

$$\frac{\partial C(Q,p)}{\partial p_i} = -\frac{\frac{V(C(Q,p),p)}{\partial p_i}}{\frac{V(C(Q,p),p)}{\partial Q}} = h_i(Q,p) = g_i(m,p) \tag{5}$$

The function $g_i(m.p)$ represents the Marshallian demand which expresses quantity demanded for an input or good as a function of its own price, the budget constraint and the price of all the other goods.

Given the Marshallian demand function of electricity the multidimensional model need to be reduced into a two dimensional problem. For this reason, all the other goods and inputs will be bundled in a numeraire good. The numeraire is evaluated at a price proxied by the monthly consumer price index (adjusted excluding from its computation the energy consumption).

3 The Statistical Model

With regard to the econometric method, the work used a Bayesian procedure, whose application in electricity demand analysis represents a novel approach.

Until recently, the Bayesian approach has been in a distinct minority in the field of econometrics, which has been dominated by the frequentist approach: computation has been the substantive reason for the minority status of Bayesian Econometrics. The computing revolution of the last twenty years has overcome this hurdle allowing to exploit the theoretical and conceptual elegance of Bayesian Statistics in the empirical studies.

The model uses a log-linear demand function: the dependent variable is the logarithm of aggregated demand and the explanatory variables are the corresponding logarithm of prices, adjusted by the monthly consumer index price (representing the price of the numeraire) and dummy variables (relative to the day the zone etc...) which approximate the total expenditure.

Analytically, the model is:

$$\log y_i = \alpha_i + \beta_i \log(\frac{p_i}{\overline{p}}) + \sum \gamma_{ki} d_{ki} \tag{6}$$

where y_i represents a point of aggregated demand and i index the hour of the day.

Given this functional form β_i represents the hourly elasticity of electricity.

Regressors d_{ki} refer both to daily and zone intercept dummies and daily and zone interaction dummies which allow to derive the hourly elasticity for each day.

Let divide the day into two groups of hours (peak and off-peak hours), one ranging from 9 a.m. to 8 p.m. (the time period in which the majority of consumption and economic activities take place), the second instead goes from 21 p.m. to 8 a.m.. We expect that participants, within these two groups of hours can affect the market price sensitivity: setting prices in advanced gives purchasers the time to react to high prices, postpone their electricity consumption, reschedule their activities and their demand profiles, flattening in this way the load curves. Given the differences in the main economics variables between peak and off-peak hours, we assume that the hourly demands and the hourly spot prices are correlated within each group. If the derived peak hour elasticities will be higher than off-peak elasticities, the assumption of economic agents conditioning market elasticity will be confirmed. On the other hand, if price responsiveness during peak hours do not significantly differs from night hour elasticities, we can conclude that purchasers have small market power and, given their stiff consumption profiles, they can not influence market equilibrium prices and quantities.

Given this market structure, we apply a Seemingly Unrelated Regression model. SUR model is a multiple equations regression model, in our case regression equations are 12, one for each hours.

The SUR can be written as:

$$y_{mi} = \beta_{m1} x_{mi1} + \beta_{m2} x_{mi2} + ... + \beta_{mik} x_{mik} + \varepsilon_{mi} \tag{7}$$

with $i = 1, ..., N$ observations for $m = 1, ..., M$ equations. M represents the number of hours whose electricity prices and loads are considered correlated). y_{mi} is the ith observation of the dependent variable (the log-demand) in equation m, x_{mik_m} (with $k = 1, ..., K$) is the ith obeservation of the of explanatory variable of the mth equation and β_{mk} is the k regression coefficient of the $m-$th equation.

Model can be written in a compact form. Let denote $y_m = (y_{m1}, ..., y_{mN})'$, $\varepsilon_m = (\varepsilon_{m1}, ..., \varepsilon_{mN})'$

$$\beta = \begin{bmatrix} \beta_1 \\ \beta_2 \\ . \\ . \\ \beta_M \end{bmatrix}$$

$$X_m = \begin{bmatrix} x'_{m1} & x'_{m2} & . & . & x'_{mk_m} \end{bmatrix}$$

and define $k = \sum_{m=1}^{M} k_m$.
Stack all vectors together as:

$$y = \begin{bmatrix} y_1 \\ y_2 \\ . \\ . \\ y_M \end{bmatrix}$$

$$\varepsilon = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ . \\ . \\ \varepsilon_M \end{bmatrix}$$

$$X = \begin{bmatrix} X_1 \\ X_2 \\ . \\ . \\ X_M \end{bmatrix}$$

the model obtained takes the following form:

$$y = X\beta + \varepsilon$$

The SUR model can be written as a familiar linear regression model. If we assume ε_{im} to be i.i.d $N(0, \sigma^2)$ for all i and all m, the model would simply turn into the normal linear regression model. However, we have assumed that market participants can reprogramming their activities and reschedule their demand profiles (within the group of hours) if they suppose changes in electricity prices. Therefore, ε_i must be i.i.d. $N(0, \Sigma_i)$ just for $i = 1, ..., N$ where Σ_i is an $M \times M$ full variance covariance matrix.

Since observations refer to different hours, heteroskedasticity can be a plausible assumption to be explored: each hour is characterized by different price volatility and variability in the load which can be included in the model.

In this work we assume:

1. For each equation the error terms ε_j, $j = 1, ..., m$ have a multivariate normal distribution with zero mean and covariance matrix Σ_j that is a positive definite matrix.

2. All elements of X remain fixed (i.e. are not random variables).

Heteroskedasticity refers to a model where the covariance matrix of the error terms are different across equations, that is, in the block diagonal matrix Ω, the non-null matrix are different from each other:

$$Var(\varepsilon) = \Omega = \begin{bmatrix} \Sigma_1 & 0 & ... & 0 \\ 0 & \Sigma_2 & ... & 0 \\ ... & ... & ... & ... \\ 0 & 0 & ... & \Sigma_m \end{bmatrix} = \begin{bmatrix} h^{-1} \times \Lambda_1 & ... & 0 \\ 0 & ... & 0 \\ ... & ... & ... \\ 0 & ... & h^{-1} \times \Lambda_m \end{bmatrix} = \Lambda^{-1} \tag{8}$$

The matrix is written in terms of precision, substituting for Σ_j the terms $h^{-1} \times \Lambda_j^{-1}$ for all $j = 1, ..., m$ where h is the precision (the inverse of the variance $\sigma^2 = h^{-1}$). Moreover, we manage a hierarchical model, since we assume that we do not know the values assumed by the elements of the Ω_i matrices. This model allows to free up the normality assumption, since unknown heteroskedasticity is equivalent to a linear regression model with Student-t errors.

Bayesian technique imposes to set Prior distributions for all parameters of interest, then, the next step will be to choose adequate prior distribution for βs parameters.

3.1 The transformed model

Before discussing the prior and posterior and the computational issues, general results of the model are presented. Since Ω is a positive definite matrix, Cholesky decomposition can be applied, then it exist a $(Nm \times Nm)$ matrix P with the property that $P\Omega P' = I_{Nm}$.

Given the model:

$$y = X\beta + \varepsilon \tag{9}$$

with $\varepsilon \sim N(0, h^{-1} \times \Omega)$

if we multiply both sides of the previous equation by P, we obtain the transformed model

$$y^* = X^*\beta + \varepsilon^* \tag{10}$$

where $y^* = Py$, $X^* = PX$ and $\varepsilon^* = P\varepsilon$. It can be verified that $\varepsilon^* \sim N(0, I_{Nm})$. Hence the transformed model falls again into the standard Normal linear regression model. There are two important implications to be discussed.

Using the properties of the multivariate Normal distribution, the likelihood function of transformed model can be seen to be:

$$\begin{aligned} p(y|\beta, h, \Lambda) &= \frac{h^{\frac{Nm}{2}}}{(2\pi)^{\frac{N \cdot m}{2}}} |\Lambda|^{\frac{1}{2}} \exp\left[-\frac{h}{2}(y - X\beta)'\Lambda(y - X\beta)\right] = \\ & \frac{h^{\frac{Nm}{2}}}{(2\pi)^{\frac{N \cdot m}{2}}} \exp\left[-\frac{h}{2}(y^* - X^*\beta)'(y^* - X^*\beta)\right] \end{aligned} \tag{11}$$

Usually Bayesian technique suggests to use natural conjugate priors, making the β's distribution be dependent Ω, in this way the joint posterior distribution would become: $p(\beta, \Omega) = p(\beta|\Omega)p(\Omega)$. This joint prior has the advantage to derive analytically tractable joint posterior distributions whose main summary statistics are available, sparing in this way the use of posterior simulator: However, the natural conjugate prior for the SUR model has been found by many to be too restrictive. The prior covariances between coefficients in each pair of equations are in fact all proportional to the same matrix. For this reason, following the mainstream literature, here I apply the extended version of the natural conjugate prior: the independent Normal - Wishart prior:

$$p(\beta) = N(\beta, V) \tag{12}$$

$$p(h) = G(\nu_0, s_0^{-2}) \tag{13}$$

Moreover, we assume that β and h have distributions independent on Ω, whose prior will be defined later.

$$p(\beta, h, \Lambda) = p(\beta)p(h)p(\Lambda) \tag{14}$$

Prior hyperparameter elicitation comes from the previous empirical study of Bigerna et al. (2014b). Then, for the beta parameters I used a Normal Prior distribution centered on the frequentist hourly estimates referring to the previous year (2010).

The joint posterior distribution of all parameters is, as always, the likelihood function times the priors:

$$\begin{aligned} p(\beta, h, \Lambda|y) \ \propto \ & p(\Lambda) \times \\ & \exp\left\{\left[-\frac{h}{2}(y^* - X^*\beta)'(y^* - X^*\beta)\right]\right\} \\ & \times \exp\left\{-\frac{1}{2}(\beta - \beta_0)'V_0^{-1}(\beta - \beta_0)\right\} \\ & \times h^{\frac{1}{2}(Nm+\nu_0-2)} \exp\left[-\frac{h\nu_0}{2s^{-2}}\right] \end{aligned} \tag{15}$$

It is not ascribable to a well-known functional form. However, functional form of the full conditional posterior distributions are known: the full conditional for β is a Normal:

$$\beta|y, h, \Lambda \sim N(\beta_n, V_n) \tag{16}$$

where:

$$V_n = (V_0^{-1} + hX\Lambda^{-1}X)^{-1} \tag{17}$$

and

$$\beta_n = V_n(V_0^{-1}\beta_0 + hX\Lambda^{-1}X\widehat{\beta}(\Lambda)) \tag{18}$$

with

$$\widehat{\beta}(\Lambda) = (X^{*\prime}X^*)^{-1}X^{*\prime}y^* = (X'\Lambda X)^{-1}X'\Lambda y \tag{19}$$

The posterior distribution of h conditional on the other parameters in the model is a Gamma:

$$h|y, \beta, \Lambda \sim G(s_n^{-2}, \nu_n) \tag{20}$$

where:

$$\nu_n = Nm + \nu_0 \tag{21}$$

and

$$s_n^2 = \frac{(y - X\beta)'\Lambda^{-1}(y - X\beta) + \nu_0 s_0^2}{\nu_n} \tag{22}$$

Conditioning on Λ, the two full conditional distributions for β and h combine data and prior information. Given Λ, the full conditional distributions of

β and h are ascribable to a well-known analytical form, as in the traditional linear model, the Gibbs Sampling algorithm can exploit this two densities and constructs a stationary Markov Chain. The first part of simulation procedure has been defined. However, the full conditional posterior of Λ does not take any recognizable form, then the joint posterior distribution $p(\beta, h, \Lambda|y)$ keeps remaining un-tractable since $p(\beta, h, \Lambda|y) \neq p(\beta, h|\Lambda, y) \cdot p(\Lambda|\beta, h, y)$. The inference related to the random parameters β, h and Λ has not been possible yet: the prior distribution for Λ needs to be investigated in order to design the second component of simulation.

When Λ is an unknown parameter, the elements of the matrix Ω in (8) become random variables. The treatment of heteroskedasticity of unknown form is a challenging task and involves the use of a hierarchical prior.

Introducing unknown heteroskedasticity increases the number of parameters to be estimated; if we treat $\Lambda_1, ..., \Lambda_m$ as completely independent and unrestricted matrices, we would not have enough observations to estimate each one of them. For this reason the exchangebility of Λ_i becomes an assumption essential to deal with this high dimensional model. The prior for Λ becomes:

$$p(\Lambda) = \prod_{j=1}^{m} f_W(\Lambda_j|\Lambda_0, \nu_\lambda) \tag{23}$$

which states that Λ_js are different from one another but they are *i.i.d.* draws from the same Wishart distribution (the hierarchical prior). The hierarchical prior imposes a structure to the model that preserves flexibility and makes estimation be possible. The use of a Wishart prior distribution allows to turn out a linear regression model with *i.i.d.* Student-t error terms with $\nu_\lambda > m$ degrees of freedom.

In other words :

$$\varepsilon_i|\Lambda_i^{-1} \sim N(0, \sigma_i^2\Lambda_i^{-1}) \tag{24}$$

$$\Lambda_i \sim W(\Lambda_0, \nu_\lambda) \tag{25}$$

$$\varepsilon_i \sim t(0, \sigma^2, \nu_\lambda) \tag{26}$$

The model becomes more flexible since Student-t distribution that is a more general class of distributions that includes Normal density as a special case (occurring when the degrees of freedom ν_λ tent to infinity).

Our treatment of unknown heteroskedasticity is equivalent to a scale mixture of Normal. The error terms ε_i are distributed according to a mixture of m different normal distributions. That is:

$$\varepsilon_i = \sum_j e_{ij} \left(\alpha_r + (H_j)^{-1/2} \eta_{ij} \right) \tag{27}$$

where η_{ij} is i.i.d $N(0, I_m)$ for $i = 1, ..., N$, $j = 1...J$ and e_{ij}, α_j and H_j are all parameters. The e_{ij} is a dichotomous random variable and indicates which distribution component in the mixture the ith error is drawn from.

$$e_{ij} = \begin{cases} 1 & \text{if } \varepsilon_j \sim N(\alpha_j, H_j) \\ 0 & \text{otherwise} \end{cases} \tag{28}$$

Since it is unknown which component the ith error is drawn from, we define $p_j = P(e_{ij} = 1)$ for $j = 1, ..., m$ the probability of the error being drawn from the jth component in the mixture. Formally it means that e_{ij} are i.i.d draws from a Multinomial distribution

$$e_i \sim M(1, p) \tag{29}$$

where $p = (p_1...p_m)'$ is the probability vector p.

The assumption that Λ_i follows a Wishart distribution and that, given Λ_i, the errors are independent Normal $(0, h^{-1}\Lambda_i^{-1})$ is equivalent to the assumption that the distribution of error term ε is a weighted average of Normals having different variances but the same means (i.e. all errors have mean equal to zero). When we mix the error terms' normal distributions using $f_W(\Lambda_i|\Lambda_0, \nu_\lambda)$, they end up to be equal to the Student-t distribution. Intuitively, assuming that a Normal model is too restrictive, a more flexible distribution taking a mixture (the weighted average) of Normals can be created. As more and more Normals are mixed, as the distribution becomes more and more flexible and can approximate any distribution with high degree of freedom. Mixtures of Normal are powerful tool to be used when economic theory does not suggest any particular form of likelihood function and you wish to be more flexible.

However, this model uses a finite mixture of Normal and it cannot be considered non-parametric in the sense that it can not accomodate any distribution, it is "just an extremely flexible modelling strategy" (Koop et al., 2007).

Parameter ν_λ is not known and Bayesian framework imposes to define a prior distribution $p(\nu_\lambda)$. The prior of λ is specified in two steps, firstly we specify $p(\Lambda|\nu_\lambda) = \prod_{i=1}^{N} f_W(\Lambda_i|\Lambda_0, \nu_\lambda)$, secondly we define $p(\nu_\lambda)$; in this way these two steps refer to a hierarchical prior model. $p(\Lambda|\nu_\lambda)$ and $p(\nu_\lambda)$ are the features necessary to design the second part of the simulation procedure.

Let it focus on $p(\Lambda|y,\beta,h,\nu_\lambda)$ and $p(\nu_\lambda|y,\beta,h,\Lambda)$.

$$p(\Lambda|y,\beta,h,\nu_\lambda) = \prod_{i=1}^{N} p(\Lambda_i|y,\beta,h,\nu_\lambda) \tag{30}$$

where

$$p(\Lambda_i|y,\beta,h,\nu_\lambda) = W\left((\nu_\lambda + m)\left[h(\varepsilon_i\varepsilon_i')\right]^{-1} + \nu_\lambda, \nu_\lambda + m\right) \tag{31}$$

Conditional on β, ε_i can be calculated and hence also the parameters of the Gamma density can be sampled within the Gibbs sampler.

Problems arise in the derivation of full conditional posterior for ν_λ. Since ν_λ is positive we assume as a prior an exponential distribution that is a gamma with two degree of freedom:

$$p(\nu_\lambda) = G(\nu_0, 2) \tag{32}$$

Then, the full conditional posterior is:

$$p(\nu_\lambda|y,\beta,h,\Lambda) = p(\nu_\lambda|\Lambda) \propto p(\Lambda|\nu_\lambda)p(\nu_\lambda)^1 \tag{33}$$

The kernel of the posterior conditional of ν_λ is simply (30) times (32):

$$\begin{aligned} p(\nu_\lambda|\Lambda) &\propto p(\Lambda|\nu_\lambda)p(\nu_\lambda) \\ &\propto \left(\frac{\nu_\lambda}{2}\right)^{\frac{N\nu_\lambda}{2}} \Gamma\left(\frac{\nu_\lambda}{2}\right)^{-N} \exp(-\eta\nu_\lambda) \end{aligned} \tag{34}$$

where

$$\eta = \frac{1}{\nu_0} + \frac{1}{2}\sum_{i=1}^{N}\left[\ln|\Lambda|_i^{-1}) + tr|\Lambda_0^{-1}\Lambda_i\right] \tag{35}$$

The density derived in (34) is not again a standard one, so algorithm for the posterior simulation of the degree of freedom need to be performed. The simulation strategy is the following:

- Use Random-Walk-Metropolis-Hastings to simulate a sample from $p(\nu_\lambda|\Lambda)$.
- Given ν_λ, run Gibbs Sampling simulating $p(\beta,h,\Lambda,\nu_\lambda|y)$ using the $\{\nu_\lambda^t\}$ sample, $p(\beta|y,h,\Lambda)$ in (16) and $p(h|y,\beta,\Lambda)$ in (20).

[1]Since ν_λ does not eneter in the likelihood $p(\nu_\lambda|y,\beta,h,\lambda) = p(\nu_\lambda|\lambda)$.

The candidate generating function is $q(\nu_\lambda^{(s-1)}; \nu_\lambda^*) = N([\nu_\lambda^* - \nu_\lambda^{(s-1)}], 0.2)$ and number of replications are set equal to 11000.

Setting the number of replication equal to 11000 guarantees that the chain takes enough steps to cover all the parameter space, diagnsotic procedure shows satisfactory results: the convergence diagnostic tests performed do not refuse the null hypotesis of convergence to the posterior density. After discarding the first 1000 realizations, the chain $\left\{\beta^i, h^i, \nu_\lambda^i\right\}_{i=1001}^{11000}$. The sequence simulate a sample from the posterior $p(\beta, h, \Lambda, \nu_\lambda|y)$.

4 Empirical Results

Data refers GME daily data for the 2011 and they had been collected in monthly dataset starting from January 2011 to December 2011. Each monthly dataset accounts for 1.5 millions of raw observations and Bid pertaining the demand side are about 400-450 thousand observations, the 20-25% of the total amount of offers. In each hour of the two years I ranked the bids according to the merit order (price descending order); I included also the rejected bids in order to have the estimation of elasticities relative to the prices of demand curve lower than equilibrium price. These latter elasticities represent in fact the real responsiveness to change in price of purchaser less incline to buy. Then, I aggregated all inelastic bids (bids with submitted price equal to 3000), computing in this way the market point of demand corresponding to the intercept. Finally I derived the remaining downward sloping market demand curve by horizontal sum of bids characterized by the same price.

At the end of the procedure each monthly Dataset accounts for a sample size ranging from 15558 observations in February to 23148 observations in November 2011. For each system of equations we derive the hourly-elasticity for each day of the month by simply adjusting the coefficient related to the log-price regressors through the coefficient related to the daily (iteration) dummy variables. In this way we have derived all beta elasticity for each hour and each day.

In order to have some statistical summaries we aggregated the later estimates in the hourly average elasticity for each month [2]. Firstly, it can be noticed that average elasticities vary within the hours of the day. In 2011 estimates go from a minimum value of from -0.14, recorded in September to -0.0360 recorded in November Moreover, allowing the variance to differ across observations of the same equations has led the peak hour elasticities to be, on average, higher than off-peak ones. The model confirms the Bollino and

[2]For a comparison with estimates derived using an Homoskedastic model see D'Errico and Bollino (2015)

Bigerna's results. Comparing to the off-peak estimates, peak hour elasticities show higher variability, going from -0.14 to -0.0484, while the off-peak ones vary between -0.070 and -0.0359. Secondly, in 2011 elasticities are higher during the off-peak hours. In the peak hours period, electricity quantities traded are greater than the average as it is shown by the previous tables and confirmed by high frequency of congestion.

[Table 1 - Table 2 here]

As regards to zone segmentation, we computed average elasticities within the group of hours having the same number of zone segmentation after congestion. Estimates shows that during Peak hours higher elasticities has in fact been recorded when the single market occurred. When the transmission constraints are violated, elasticity becomes lower and this is particular evident during the peak hours. High levels of demand causing congestion reduces the responsiveness to change in price. Moreover, frequent congestions during peak hour may suggest that electricity is an essential commodity whose demand is stiff and whose consumption can not be postponed.

Off-peak estimates instead, do not show a well defined behaviour. During off-peak hours electricity is allocated essentially for domestic uses and it means that in this model end-consumers have take the same behaviour with respect to market segmentation and the risk of congestion.

Elasticities aggregated by PUN percentiles are higher when lower levels of price have been recorded. Lower price levels mean low quantities traded and lower income levels. Then, as I said before, consumers with limited expenditure availability (referring essentially to domestic user) have more flexible behaviour given changes in price.

[Table 3 - Table 5 here]

The decomposition of average elasticities between peak and off-peak hours shows that within peak hours, the lower elasticities were recorded when both single market and maximum segmentation market (four zone division) occurred. The lowest average elasticity was recorded in the presence of maximum segmentation of national market that gives evidence of higher levels of demand and income. As we said before, high levels of expenditure availability affect demand elasticity reducing the responsiveness to change in price. Moreover, congestions during the peak hours may suggest that electricity is an essential commodity whose demand is stiff and whose consumption can not be postponed.

[Table 6 - Table 10 here]

5 Conclusions

The models proposed highlights that buyers in the Italian Wholesale Electricity Market react to change in price since the estimated elasticities are different from zero.

Moreover, the estimates differ from one another during the day, on the strenght of the level of electricity loads, the market segmentation structure and the levels of PUN.

In the Heteroskedastic Multivariate Linear Model, the elasticities recorded during peak hours are higher than Off-Peak estimates. Moreover buyers mantain their higher reactivity to changes in price when there is not congestion and when PUN records low values.

Further development of the research may be the application of the heteroskedastic model to the more recent data (from 2012 to 2015). Moreover, also the computational part can be implemented. Heteroskedastic model represents a novel in the empirical analyses, but the multi-dimension of the inferential problem made the construction of the algorithm the most challenging task of my thesis. The heteroskedastic model designs for empirical data a statistical framework rigorous and detailed. However, posterior simulation requires the discretional choice of the proposal density and the tuning of its parameters, first of all the variance, affecting the behaviour of the chain and the resulting posterior. Running the procedure with other parameters and alternative functional forms of the candidate generating density could be a possible development in order to make a comparison between the different derived estimates.

6 Tables

	Hour	January	February	March	April	May	June	July	August	September	October	November	December
Peak	9	-0.0756	-0.0699	-0.0869	-0.0676	-0.0853	-0.0495	-0.0506	-0.0730	-0.1388	-0.0579	-0.0603	-0.0599
	10	-0.0764	-0.0696	-0.0864	-0.0679	-0.0855	-0.0492	-0.0509	-0.0731	-0.1321	-0.0582	-0.0598	-0.0600
	11	-0.0771	-0.0695	-0.0867	-0.0682	-0.0859	-0.0484	-0.0510	-0.0769	-0.1387	-0.0582	-0.0599	-0.0579
	12	-0.0771	-0.0705	-0.0872	-0.0682	-0.0863	-0.0485	-0.0504	-0.0728	-0.1342	-0.0578	-0.0600	-0.0556
	13	-0.0769	-0.0707	-0.0877	-0.0677	-0.0868	-0.0490	-0.0505	-0.0754	-0.1355	-0.0568	-0.0599	-0.0571
	14	-0.0761	-0.0701	-0.0876	-0.0686	-0.0863	-0.0493	-0.0503	-0.0777	-0.1416	-0.0569	-0.0603	-0.0579
	15	-0.0759	-0.0701	-0.0878	-0.0677	-0.0859	-0.0498	-0.0505	-0.0758	-0.1374	-0.0571	-0.0607	-0.0615
	16	-0.0758	-0.0706	-0.0875	-0.0682	-0.0852	-0.0496	-0.0508	-0.0756	-0.1398	-0.0578	-0.0602	-0.0588
	17	-0.0751	-0.0707	-0.0872	-0.0678	-0.0853	-0.0488	-0.0512	-0.0757	-0.1371	-0.0579	-0.0605	-0.0610
	18	-0.0751	-0.0706	-0.0868	-0.0676	-0.0859	-0.0486	-0.0514	-0.0767	-0.1369	-0.0578	-0.0611	-0.0612
	19	-0.0759	-0.0703	-0.0869	-0.0679	-0.0856	-0.0493	-0.0520	-0.0795	-0.1387	-0.0586	-0.0610	-0.0613
	20	-0.0753	-0.0710	-0.0869	-0.0688	-0.0856	-0.0499	-0.0516	-0.0782	-0.1434	-0.0582	-0.0611	-0.0581
Off-Peak	21	-0.0571	-0.0528	-0.0452	-0.0580	-0.0630	-0.0438	-0.0594	-0.0577	-0.0498	-0.0707	-0.0395	-0.0607
	22	-0.0572	-0.0541	-0.0456	-0.0592	-0.0657	-0.0434	-0.0588	-0.0576	-0.0487	-0.0706	-0.0419	-0.0612
	23	-0.0573	-0.0556	-0.0489	-0.0580	-0.0629	-0.0432	-0.0592	-0.0576	-0.0495	-0.0676	-0.0447	-0.0596
	24	-0.0576	-0.0522	-0.0459	-0.0613	-0.0662	-0.0427	-0.0588	-0.0577	-0.0498	-0.0664	-0.0418	-0.0595
	1	-0.0576	-0.0553	-0.0455	-0.0595	-0.0650	-0.0432	-0.0587	-0.0578	-0.0496	-0.0701	-0.0422	-0.0601
	2	-0.0581	-0.0557	-0.0470	-0.0603	-0.0679	-0.0439	-0.0588	-0.0578	-0.0503	-0.0675	-0.0460	-0.0599
	3	-0.0580	-0.0559	-0.0504	-0.0619	-0.0632	-0.0438	-0.0588	-0.0577	-0.0514	-0.0645	-0.0372	-0.0600
	4	-0.0579	-0.0556	-0.0521	-0.0641	-0.0642	-0.0439	-0.0590	-0.0577	-0.0506	-0.0668	-0.0365	-0.0604
	5	-0.0573	-0.0532	-0.0526	-0.0594	-0.0665	-0.0429	-0.0593	-0.0569	-0.0504	-0.0622	-0.0359	-0.0607
	6	-0.0567	-0.0519	-0.0517	-0.0584	-0.0635	-0.0429	-0.0591	-0.0569	-0.0510	-0.0675	-0.0362	-0.0603
	7	-0.0570	-0.0531	-0.0500	-0.0582	-0.0666	-0.0435	-0.0593	-0.0569	-0.0509	-0.0689	-0.0360	-0.0609
	8	-0.0572	-0.0526	-0.0475	-0.0624	-0.0676	-0.0441	-0.0591	-0.0565	-0.0513	-0.0675	-0.0366	-0.0611

Tab. 1: Hourly Average Elasticity. 2011.

Average Elasticity	2011			
	I Quarter	II Quarter	III Quarter	IV Quarter
Peak	-0.0778	-0.0677	-0.0882	-0.0591
Off-Peak	-0.0533	-0.0562	-0.0556	-0.0558

Tab. 2: Average Elasticity by Quarter. 2011.

Percentile	January		February		March		April		May		Jun	
	Peak	Off-Peak	Peak	Off-Peak	Peak	Off-Peak	Peak	Off-Peak	Peak	Off-Peak	Peak	Off-Peak
10	-0.0703	.	-0.0710	.	-0.0858	-0.0723	-0.0656	-0.0438	-0.0832	-0.0742	-0.0520	-0.0574
9	-0.0738	.	-0.0694	-0.0374	-0.0861	-0.0502	-0.0695	-0.0664	-0.0878	-0.0587	-0.0518	-0.0439
8	-0.0742	-0.0692	-0.0708	-0.0630	-0.0854	-0.0406	-0.0709	-0.0613	-0.0859	-0.0715	-0.0497	-0.0447
7	-0.0762	-0.0664	-0.0685	-0.0508	-0.0877	-0.0597	-0.0685	-0.0714	-0.0883	-0.0731	-0.0481	-0.0447
6	-0.0796	-0.0560	-0.0700	-0.0512	-0.0862	-0.0594	-0.0646	-0.0587	-0.0859	-0.0682	-0.0485	-0.0473
5	-0.0761	-0.0542	-0.0697	-0.0558	-0.0880	-0.0299	-0.0695	-0.0600	-0.0826	-0.0729	-0.0514	-0.0466
4	-0.0763	-0.0552	-0.0704	-0.0522	-0.0887	-0.0430	-0.0666	-0.0566	-0.0836	-0.0585	-0.0456	-0.0404
3	-0.0718	-0.0575	-0.0729	-0.0565	-0.0902	-0.0499	-0.0670	-0.0607	-0.0842	-0.0633	-0.0496	-0.0455
2	-0.0716	-0.0613	-0.0753	-0.0498	-0.0892	-0.0535	-0.0719	-0.0619	-0.0812	-0.0657	-0.0457	-0.0403
1	.	-0.0547	.	-0.0568	.	-0.0478	-0.0713	-0.0590	-0.0799	-0.0570	-0.0394	-0.0435

Tab. 3: Monthly Peak Average Elsticity by PUN Percentile. Jan.-Jun.2011

Percentile	July		August		September		October		November		December	
	Peak	Off-Peak	Peak	Off-Peak	Peak	Off-Peak	Peak	Off-Peak	Peak	Off-Peak	Peak	Off-Peak
10	-0.0505	-0.0648	-0.0840	-0.0480	-0.1652	-0.0498	-0.0577	-0.0715	-0.0609	-0.0384	-0.0620	-0.0618
9	-0.0531	-0.0685	-0.0840	-0.0612	-0.1211	-0.0501	-0.0570	-0.0614	-0.0608	-0.0449	-0.0600	-0.0620
8	-0.0490	-0.0604	-0.0721	-0.0588	-0.1273	-0.0552	-0.0586	-0.0766	-0.0589	-0.0305	-0.0536	-0.0583
7	-0.0522	-0.0578	-0.0749	-0.0565	-0.1157	-0.0483	-0.0574	-0.0651	-0.0608	-0.0470	-0.0481	-0.0612
6	-0.0513	-0.0613	-0.0732	-0.0533	-0.0808	-0.0443	-0.0571	-0.0696	-0.0588	-0.0412	-0.0630	-0.0566
5	-0.0511	-0.0654	-0.0736	-0.0563	-0.1743	-0.0480	-0.0601	-0.0668	-0.0585	-0.0426	-0.0641	-0.0644
4	-0.0494	-0.0603	-0.0715	-0.0618	-0.1503	-0.0507	-0.0593	-0.0638	-0.0621	-0.0385	-0.0636	-0.0699
3	-0.0516	-0.0581	-0.0777	-0.0556	-0.1614	-0.0511	-0.0564	-0.0562	-0.0617	-0.0517	-0.0633	-0.0663
2	-0.0468	-0.0605	-0.0668	-0.0636	-0.1293	-0.0564	-0.0612	-0.0666	-0.0632	-0.0434	-0.0917	-0.0533
1	-0.0552	-0.0526	-0.0692	-0.0599	.	-0.0541	-0.0630	-0.0759	-0.0640	-0.0299	.	-0.0608

Tab. 4: Monthly Average Elsticity by PUN Percentile. Jul.-Dec.2011

Percentile	Mean		Stand. Dev		Min.		Max.	
	Peak	Off-Peak	Peak	Off-Peak	Peak	Off-Peak	Peak	Off-Peak
1	-0.070	-0.054	0.023	0.028	-0.224	-0.131	-0.010	0.000
2	-0.072	-0.052	0.019	0.027	-0.216	-0.130	-0.010	0.000
3	-0.070	-0.053	0.022	0.025	-0.262	-0.125	-0.012	-0.001
4	-0.068	-0.059	0.021	0.024	-0.217	-0.132	-0.001	-0.002
5	-0.070	-0.055	0.027	0.027	-0.292	-0.126	-0.005	0.000
6	-0.070	-0.054	0.032	0.026	-0.291	-0.127	0.000	0.000
7	-0.073	-0.057	0.037	0.024	-0.252	-0.129	0.000	0.000
8	-0.078	-0.056	0.045	0.024	-0.282	-0.124	-0.001	0.000
9	-0.078	-0.057	0.049	0.024	-0.298	-0.119	-0.001	-0.001
10	-0.084	-0.056	0.060	0.024	-0.309	-0.099	-0.002	0.000

Tab. 5: Average Elsticity by PUN Percentile. 2011

Zone	2011-Jan.-Jun.			2011-Jul.-Dec.		
	Average	Peak	Off-Peak	Average	Peak	Off-Peak
5	0	0	0	0	0	0
4	-0.062	-0.068	-0.056	-0.061	-0.057	-0.065
3	-0.060	-0.067	-0.053	-0.067	-0.076	-0.058
2	-0.063	-0.069	-0.056	-0.065	-0.071	-0.059
1	-0.062	-0.070	-0.054	-0.062	-0.070	-0.055

Tab. 6: Average Elasticity by Zone Segmentation. 2011.

Zone	January		February		March		April		May		June	
	Freq.	Av. Elast.	Freq.	Av. Elast.	Freq.	Av. Elast.	Freq.	Av. Elast.	Freq.	Av. Elast.	Freq.	Av. Elast.
5	0	.	0	.	0	.	0	.	0	.	0	.
4	5	-0.077	11	-0.061	4	-0.091	1	-0.065	16	-0.079	17	-0.035
3	91	-0.077	146	-0.061	125	-0.085	110	-0.067	107	-0.084	133	-0.029
2	269	-0.075	178	-0.060	226	-0.088	215	-0.068	225	-0.088	205	-0.034
1	7	-0.077	1	-0.059	17	-0.090	34	-0.071	24	-0.081	5	-0.043

Tab. 7: Average Peak Elasticity by Zone Segmentation. Jan.-Jun. 2011.

Zone	July		August		September		October		November		December	
	Freq.	Av. Elast.	Freq.	Av. Elast.	Freq.	Av. Elast.	Freq.	Av. Elast.	Freq.	Av. Elast.	Freq.	Av. Elast.
5	0	.	0	.	0	.	0	.	0	.	0	.
4	22	-0.065	0	.	10	-0.047	7	-0.056	1	-0.061	2	-0.054
3	152	-0.060	52	-0.067	221	-0.147	146	-0.060	163	-0.061	137	-0.060
2	196	-0.062	311	-0.060	124	-0.132	202	-0.056	156	-0.060	202	-0.059
1	2	-0.076	9	-0.088	5	-0.074	17	-0.064	40	-0.059	31	-0.057

Tab. 8: Average Peak Elasticity by Zone Segmentation. Jul.-Dec. 2011.

Zone	January		February		March		April		May		June	
	Freq.	Av. Elast.	Freq.	Av. Elast.	Freq.	Av. Elast.	Freq.	Av. Elast.	Freq.	Av. Elast.	Freq.	Av. Elast.
5	0	.	0	.	0	.	0	.	0	.	0	.
4	7	-0.055	10	-0.048	3	-0.070	8	-0.058	15	-0.067	18	-0.040
3	117	-0.057	73	-0.052	105	-0.053	152	-0.060	162	-0.061	123	-0.035
2	121	-0.057	135	-0.057	122	-0.042	120	-0.061	165	-0.070	157	-0.051
1	127	-0.058	118	-0.053	141	-0.050	80	-0.060	30	-0.061	62	-0.043

Tab. 9: Average Off-Peak Elasticity by Zone Segmentation. Jan.-Jun. 2011.

Zone	July		August		September		October		November		December	
	Freq.	Av. Elast.	Freq.	Av. Elast.	Freq.	Av. Elast.	Freq.	Av. Elast.	Freq.	Av. Elast.	Freq.	Av Elast.
5	0	.	0	.	0	.	0	.	0	.	0	.
4	25	-0.057	5	-0.079	7	-0.059	7	-0.067	1	-0.079	4	-0.048
3	193	-0.058	162	-0.057	172	-0.054	93	-0.066	94	-0.057	67	-0.052
2	109	-0.060	128	-0.060	99	-0.047	155	-0.069	127	-0.060	153	-0.057
1	45	-0.062	77	-0.051	82	-0.046	117	-0.067	138	-0.051	148	-0.053

Tab. 10: Average Off-Peak Elasticity by Zone Segmentation. Jul.-Dec. 2011.

References

Aalami, H., Moghaddam, M. P., and Yousefi, G. (2010). Modeling and prioritizing demand response programs in power markets. *Electric Power Systems Research*, 80(4):426–435.

Albadi, M. H. and El-Saadany, E. (2008). A summary of demand response in electricity markets. *Electric power systems research*, 78(11):1989–1996.

Ando, T. (2012). Bayesian variable selection for the seemingly unrelated regression models with a large number of predictors. *Journal of the Japan Statistical Society*, 41(2):187–203.

Ando, T. and Zellner, A. (2010). Hierarchical bayesian analysis of the seemingly unrelated regression and simultaneous equations models using a combination of direct monte carlo and importance sampling techniques. *Bayesian Anal.*, 5(1):65–95.

B. Ravikumar, Surajit Ray, N. E. S. (2000). Robust wald tests in sur systems with adding-up restrictions. *Econometrica*, 68(3):715–719.

Berndt, E. R. and Wood, D. O. (1975). Technology, prices, and the derived demand for energy. *The review of Economics and Statistics*, pages 259–268.

Bigerna, S., Bollino, C. A., et al. (2014a). Electricity demand in wholesale italian market. *Energy J*, 35:25–46.

Bigerna, S., Bollino, C. A., and Polinori, P. (2014b). Market power and transmission congestion in the italian electricity market.

Borenstein, S., Bushnell, J., and Wolak, F. (2002). Measuring market inefficiencies in californias restructured wholesale electricity market. csem wp-102. *Center for the Study of Energy Markets, University of California Energy Institute*.

Cardell, J. B., Hitt, C. C., and Hogan, W. W. (1997). Market power and strategic interaction in electricity networks. *Resource and energy economics*, 19(1):109–137.

Chib, S. and Greenberg, E. (1995a). Hierarchical analysis of sur models with extensions to correlated serial errors and time-varying parameter models. *Journal of Econometrics*, 68(2):339–360.

Chib, S. and Greenberg, E. (1995b). Understanding the metropolis-hastings algorithm. *The american statistician*, 49(4):327–335.

Creel, M. and Farell, M. (1996). Sur estimation of multiple time-series models with heteroscedasticity and serial correlation of unknown form. *Economics letters*, 53(3):239–245.

D'Errico, M. C. and Bollino, C. A. (2015). Bayesian analysis of demand elasticity in the italian electricity market. *Sustainability*, 7(9):12127–12148.

Gelfand, A. E. and Smith, A. F. (1990). Sampling-based approaches to calculating marginal densities. *Journal of the American statistical association*, 85(410):398–409.

Gelman, A., Roberts, G., and Gilks, W. (1996). Efficient metropolis jumping hules. *Bayesian statistics*, 5(599-608):42.

Geweke, J. (1989). Bayesian inference in econometric models using monte carlo integration. *Econometrica: Journal of the Econometric Society*, pages 1317–1339.

Gilks, W. R. (2005). *Markov chain monte carlo*. Wiley Online Library.

Hastings, W. K. (1970). Monte carlo sampling methods using markov chains and their applications. *Biometrika*, 57(1):97–109.

Kahn, E. P. (1998). Numerical techniques for analyzing market power in electricity. *The Electricity Journal*, 11(6):34–43.

Koop, G., Poirier, D. J., and Tobias, J. L. (2007). *Bayesian econometric methods*. Cambridge University Press.

Lo Prete, C. and Hobbs, B. F. (2015). Market power in power markets: An analysis of residual demand curves in california's day-ahead energy market (1998-2000). *Energy Journal*, 36(2).

Mei, B. and Sun, C. (2008). Assessing time-varying oligopoly and oligopsony power in the us paper industry. *Journal of Agricultural and Applied Economics*, 40(03):927–939.

Muhammad, A. et al. (2006). Measuring the degree of monopsony power in the eu fish importing industry: Implications for ugandan fresh and chilled fish fillet exports. In *Selected Paper prepared for presentation at the Southern Agricultural Economics Association Annual Meetings Orlando, Florida*.

Muratori, M., Schuelke-Leech, B.-A., and Rizzoni, G. (2014). Role of residential demand response in modern electricity markets. *Renewable and Sustainable Energy Reviews*, 33:546–553.

Murray, B. C. (1995). Measuring oligopsony power with shadow prices: Us markets for pulpwood and sawlogs. *The Review of Economics and Statistics*, pages 486–498.

Sweeting, A. (2007). Market power in the england and wales wholesale electricity market 1995–2000*. *The Economic Journal*, 117(520):654–685.

Taylor, L. D. (1975). The demand for electricity: a survey. *The Bell Journal of Economics*, pages 74–110.

Wolak, F. (2009). Report on market performance and market monitoring in the colombian electricity supply industry. *Department of Economics, Stanford University*.

Wolak, F. A. (2000). An empirical analysis of the impact of hedge contracts on bidding behavior in a competitive electricity market*. *International Economic Journal*, 14(2):1–39.

Wolak, F. A. (2001). Identification and estimation of cost functions using observed bid data: an application to electricity markets. Technical report, National Bureau of Economic Research.

Wolak, F. A. (2003). Measuring unilateral market power in wholesale electricity markets: the california market, 1998-2000. *American Economic Review*, pages 425–430.

Zellner, A. and Ando, T. (2010). A direct monte carlo approach for bayesian analysis of the seemingly unrelated regression model. *Journal of Econometrics*, 159(1):33–45.

1 The Italian Power Exchange

Electricity industry is a leading industrial sector since it is a fundamental input for the production processes in any industrialised country. Its strategic importance for economic development and its social and environmental impact imposes an effective regulation. For this reason it is not surprising that the electric sector was regulated by public commissions and the tariffs were kept fixed over long periods of time.

In the last decades liberalization process started in most of the developed countries, the ownership in the electricity sector became private and industry has been split up into the different functions.

The liberalization of the electricity sector has led to overcome the system of vertically integrated monopoly. Generation and retail functions have become open to competition.

Transition from state-owned monopolies to competitive markets was not always smooth and concerns had been raising in many countries; market structure affects in fact competition and for this reason the design of deregulated electricity markets offer economists a changeling opportunity. They have been attempting to design well functioning markets that gives players the correct incentives to improve production efficiency and limit market power. In the recent years many economists have focused on the effects that market design may have on equilibrium prices market power of supplier. The market structure affects in fact the consumer reactivity to change in price, that is the elasticity.

As in other international experiences, the creation of the Italian Electricity market (IPEX) responded to two specific requirements:

• promoting competition in electricity generation, sale and purchase, under criteria of neutrality, transparency and objectivity, through the creation of a market place;

• ensuring the economic management of an adequate availability of ancillary services.

The organization and the management of the Italian electricity market has been entrusted GME. Unlike other European markets, Italian Power Exchange is not a purely financial market aimed only to the definition of prices and quantities, but it is a physical market where injection and withdrawal profiles are scheduled and really delivered.

The Electricity Market is articulated in the Spot Electricity Market (MPE), Forward Electricity Market and the Financial Derivatives Market (IDEX). The Spot Electricity Market is divided into three submarkets:

The **Day-Ahead Market (MGP)**, which is the venue for the trading of electricity supply offers and demand bids for each hour of the next day.

All electricity operators may participate in the MGP. GME accepts Offers and Bids by the merit order, taking into account the current transmission constraints. Accepted supply offers are remunerated at the Zonal Clearing Price, while accepted demand bids are remunerated at the National Single Price (PUN). The accepted Offers/Bids determine the preliminary Injection and Withdrawal Schedules of each Offer Point for the next day.

The **Intra-Day Market (MI)**, which has replaced the existing Adjustment Market, it is venue for the trading of electricity supply offers and demand bids which modify the Injection and Withdrawal Schedules resulting from the Day-Ahead Market. GME accepts the Offers and Bids submitted into the MI by merit order, taking into account the Transmission Limits remaining after the Day-Ahead Market. Accepted Offers and Bids are remunerated at the Zonal Clearing Price and they Bids modify the preliminary schedules determining the revised injection and withdrawal schedules for the next day.

The **Ancillary Services Market (MSD)** is the venue for the trading of supply offers and demand bids in respect of ancillary services. This market is essentially used to acquire resources for relieving intra-zonal congestions, procuring Reserve Capacity and balancing the injections and withdrawals in the real time. Participation in the MSD is restricted to units that are authorised to supply ancillary services and to their dispatching users. Participation in the MSD is mandatory.

We focus on the Day-Ahead Market (MGP) where hourly blocks of electricity are traded for the next day are negotiated. In this market both the injection and withdrawal programs for the next day are defined in order to reach the equilibrium prices and quantities. The MGP is organized according to an implicit double auction model and the most of the transactions takes place in this market. The session opens at 8 a.m. on the ninth day before the delivery-day and closes at 9.15 a.m. on the day before the delivery is executed.

During the session, market participants submit offers to buy or sell that indicate the amount of energy and the maximum price (or the lowest price) at which they are willing to buy (or sell). In particular:

- The offers to buy (BID) represent the willingness to purchase an amount of energy that does not exceed that specified in the offer at a price no higher than that reported in the same offer.
- The offers to sell (OFF) express instead the willingness to sell an amount of energy not greater than that specified in the offer and at a price not lower than that indicated in the same offer. In the supply

side operators can relate offers only to the injection points. If the offer is accepted, the producer undertakes to enter in the network, in a given period, the amount of electricity specified in the offer.

Each offer, to sale and purchase, must be consistent with the physical constraints of the corresponding unit point. The Day-Ahead Market is a zonal market, reflecting the structure which the national transmission grid is divided in. Each zone is characterized by an insufficient interconnection capacity and when a congestion occurs the selling price is zonal differentiated: selling price is lower in the upstream area of congestion and higher in the downstream ones. In depth, when the market session closes, the GME starts the process for the resolution of the market. For each hour of the next day, the algorithm accepts all the bids and offers in order to maximize the value of trading, within the limits of maximum transit between zones.

The process of acceptance can be summarized as follows:

All offers to sell are sorted according an ascending price order forming aggregate supply curve,while bids are ordered by descending price order drawing the aggregate demand curve.

The intersection between the two curves derives the total quantity traded, the equilibrium price, the accepted BID and OFF.

If electricity flows resulting from the programs do not violate any transition constraints, the equilibrium price is unique for all the zones. The accepted offers to sale are those whose sale prices are not higher than the equilibrium price, while the accepted bids are those whose purchase prices is not lower than the equilibrium price.

If at least one transmission constraint is violated, sale price are zonal differentiated and the algorithm starts the so called "Market Splitting Mechanism". It splits in fact the market into two zones, one for the export, which includes all zones upstream of the bond, and one for the import, which includes all areas downstream of the bond, repeating in each of the two areas the process described above: i.e. it derives in each zone the corresponding aggregate supply and demand curve. The outcome are two equilibrium zonal price zone (p_{z_1} and p_{z_2}). In particular, p_{z_i} is greater in the area of import and is smaller in the area of export. If, within each zone, the resulting equilibrium quantities violate further transition constraints, the splitting market process goes on within the zones in order to obtain an outcome consistent with the grid constraints.

With regard to the purchase price of electricity, GME has implemented an algorithm that, given congestion and differentiated zonal sale prices, apply just a single national purchase price (PUN), that is the average of the zonal sale prices weighted with the zonal consumptions. The PUN applies only to withdrawal points belonging to national geographical areas.

The mechanism of market splitting is an "implicit auction" for the non-discriminatory allocation of the transit rights.

2 Theoretical Background

Since the early 1970s, when energy caught the attention of policy makers in the aftermath of the first oil crisis, research on energy demand has vastly increased in order to overcome the limited understanding of the nature of energy demand and demand response due to the presence of external shocks encountered at that time.

Elasticity, in the energy demand analysis framework, is a feature that has received particular attention in the studies of consumer preference and willingness to pay, as in the institutional studies guiding policy decisions as taxation and welfare. Moreover, the consumer reactivity to changes in price can express market efficiency. Then, in strategic economic sectors, this measure can be seen as a tool leading the National Regulators in the market structure definition processes. Previous empirical studies used data referring the supply side of electricity market, given the assumption of oligopolistic market structure, they estimate demand elasticity using residual demand function. Bigerna and Bollino's work [7] has been the first Italian study of electricity demand elasticity using data referring the demand side.

Following this approach, this work estimated demand elasticity using the same type of data. The main participants in the Italian Electricity wholesale Market are industrial consumers using power as an input in the production function to produce goods and services, while residential consumers have a domestic use of electricity. Industrial agents choose the amount of electricity input which minimizes their cost function given the technological constraint, while residential customers are part of optimizing utility function process given the budget constraint. For this reason our econometric approach will lie inside the neoclassical framework and will be grounded on rational optimizing behaviour theory.

Although data available refers only market prices and demand, the duality approach gives us a theoretical justification, allowing to legitimately switch from agent's preference (optimization theory) to market demand (The Marshallian demand) in which quantities are functions of prices and total expenditure. We assume all the agent taking part in the MGP rationally behave minimizing a cost function, (production cost function for industrial buyers and expenditure function for the residential ones).

Recalling the tradition introduced by Brendt and Wood [8] the cost function assumed is the trans-log cost function, that is the the second order approximation of an agent's cost function. Its general form can be written as follow:

$$\begin{aligned}\ln C = \; & \alpha_0 + \sum \alpha_i \ln p_i + \tfrac{1}{2} \sum_i \sum_j \gamma_{ij} \ln p_i \ln p_j + \alpha_Q \ln Q \\ & + \tfrac{1}{2}\gamma_{QQ}(\ln Q)^2 + \sum_i \gamma_{Q_i} \ln Q \ln p_i \end{aligned} \tag{1}$$

where C is the total cost, i and j are the inputs (for industrial consumers) or the other good for residential customers, p_i is the factor or good prices, Q is the objective variable (the objective variable to be maximized: it can be the output quantity or the consumer's utility)

This cost function must satisfies certain properties:

- Homogeneous of degree 1 in prices;
- Satisfying all the conditions guaranteeing a well-behaved production (or utility) function
- Homothetic (separable function of the objective variable and prices).

Minimization problem is usually solved using Lagrangian techniques, leading to the first order condition:

$$\frac{\partial C(Q,p)}{\partial p_i} = h_i(Q,p) = q_i \text{ for all } i \tag{2}$$

Under the given assumptions, solving the problem yields to a demand functions expressed in terms of prices and the objective variable: $q_i = h_i(Q,p)$. These functions are the Hicksian demands or the compensated demand equations because they consider the objective variable Q as a constant parameter. For empirical works the optimization model need to be linked to economical model in which quantities are a function of prices and total expenditure. The duality approach is the theoretical framework allowing to shift from the production possibility sets (and the system of preferences) to the market demand function.

Given the convexity of production possibility sets (or convex preferences for end consumers), the Roy Identity allows to derive Marshallian demand from the Hicksian demand substituting the objective variable Q in the Hicksian demand with the profit function (or the indirect utility function).

First we derive the Minimum Expenditure function and we put it into profit function or the indirect utility function $V(m,p)$, substituting m with $C(Q,p)$ evaluated at the optimum level. This lead to the trivial identity:

$$V(C(Q,p),p) = Q(m,p) \tag{3}$$

where $Q(m,p)$ is the utility/profit function of the maximization problem, p is the price vector and m is the budget constraint. This says that the indirect profit/utility function $V(C(Q,p),p)$, that minimizes the cost for achieving a given level of utility given a set of prices, is equal to that utility function u (of the maximization problem) evaluated at those prices . Taking the derivative of both sides of this equation with respect to the price of a single input/good p_i (with the Q's level held constant) gives:

$$\frac{V(C(Q,p),p)}{\partial Q} \cdot \frac{\partial C(Q,p)}{\partial p_i} + \frac{V(C(Q,p),p)}{\partial p_i} = 0 \tag{4}$$

Rearranging what we obtain is:

$$\frac{\partial C(Q,p)}{\partial p_i} = -\frac{\frac{V(C(Q,p),p)}{\partial p_i}}{\frac{V(C(Q,p),p)}{\partial Q}} = h_i(Q,p) = g_i(m,p) \tag{5}$$

The function $g_i(m.p)$ represents the Marshallian demand which expresses quantity demanded for an input or good as a function of its own price, the budget constraint and the price of all the other goods.

Given the Marshallian demand function of electricity the multidimensional model need to be reduced into a two dimensional problem. For this reason, all the other goods and inputs will be bundled in a numeraire good. The numeraire is evaluated at a price proxied by the monthly consumer price index (adjusted excluding from its computation the energy consumption).

3 The Statistical Model

With regard to the econometric method, the work used a Bayesian procedure, whose application in electricity demand analysis represents a novel approach.

Until recently, the Bayesian approach has been in a distinct minority in the field of econometrics, which has been dominated by the frequentist approach: computation has been the substantive reason for the minority status of Bayesian Econometrics. The computing revolution of the last twenty years has overcome this hurdle allowing to exploit the theoretical and conceptual elegance of Bayesian Statistics in the empirical studies.

The model uses a log-linear demand function: the dependent variable is the logarithm of aggregated demand and the explanatory variables are the corresponding logarithm of prices, adjusted by the monthly consumer index price (representing the price of the numeraire) and dummy variables (relative to the day the zone etc...) which approximate the total expenditure.

Analytically, the model is:

$$\log y_i = \alpha_i + \beta_i \log(\frac{p_i}{\overline{p}}) + \sum \gamma_{ki} d_{ki} \tag{6}$$

where y_i represents a point of aggregated demand and i index the hour of the day.

Given this functional form β_i represents the hourly elasticity of electricity.

Regressors d_{ki} refer both to daily and zone intercept dummies and daily and zone interaction dummies which allow to derive the hourly elasticity for each day.

Let divide the day into two groups of hours (peak and off-peak hours), one ranging from 9 a.m. to 8 p.m. (the time period in which the majority of consumption and economic activities take place), the second instead goes from 21 p.m. to 8 a.m.. We expect that participants, within these two groups of hours can affect the market price sensitivity: setting prices in advanced gives purchasers the time to react to high prices, postpone their electricity consumption, reschedule their activities and their demand profiles, flattening in this way the load curves. Given the differences in the main economics variables between peak and off-peak hours, we assume that the hourly demands and the hourly spot prices are correlated within each group. If the derived peak hour elasticities will be higher than off-peak elasticities, the assumption of economic agents conditioning market elasticity will be confirmed. On the other hand, if price responsiveness during peak hours do not significantly differs from night hour elasticities, we can conclude that purchasers have small market power and, given their stiff consumption profiles, they can not influence market equilibrium prices and quantities.

Given this market structure, we apply a Seemingly Unrelated Regression model. SUR model is a multiple equations regression model, in our case regression equations are 12, one for each hours.

The SUR can be written as:

$$y_{mi} = \beta_{m1} x_{mi1} + \beta_{m2} x_{mi2} + ... + \beta_{mik} x_{mik} + \varepsilon_{mi} \tag{7}$$

with $i = 1, ..., N$ observations for $m = 1, ..., M$ equations. M represents the number of hours whose electricity prices and loads are considered correlated). y_{mi} is the ith observation of the dependent variable (the log-demand) in equation m, x_{mik_m} (with $k = 1, ..., K$) is the ith obeservation of the of explanatory variable of the mth equation and β_{mk} is the k regression coefficient of the $m-$th equation.

Model can be written in a compact form. Let denote $y_m = (y_{m1}, ..., y_{mN})'$, $\varepsilon_m = (\varepsilon_{m1}, ..., \varepsilon_{mN})'$

$$\beta = \begin{bmatrix} \beta_1 \\ \beta_2 \\ . \\ . \\ \beta_M \end{bmatrix}$$

$$X_m = \begin{bmatrix} x'_{m1} & x'_{m2} & . & . & x'_{mk_m} \end{bmatrix}$$

and define $k = \sum_{m=1}^{M} k_m$.
Stack all vectors together as:

$$y = \begin{bmatrix} y_1 \\ y_2 \\ . \\ . \\ y_M \end{bmatrix}$$

$$\varepsilon = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ . \\ . \\ \varepsilon_M \end{bmatrix}$$

$$X = \begin{bmatrix} X_1 \\ X_2 \\ . \\ . \\ X_M \end{bmatrix}$$

the model obtained takes the following form:

$$y = X\beta + \varepsilon$$

The SUR model can be written as a familiar linear regression model. If we assume ε_{im} to be i.i.d $N(0, \sigma^2)$ for all i and all m, the model would simply turn into the normal linear regression model. However, we have assumed that market participants can reprogramming their activities and reschedule their demand profiles (within the group of hours) if they suppose changes in electricity prices. Therefore, ε_i must be i.i.d. $N(0, \Sigma_i)$ just for $i = 1, ..., N$ where Σ_i is an $M \times M$ full variance covariance matrix.

Since observations refer to different hours, heteroskedasticity can be a plausible assumption to be explored: each hour is characterized by different price volatility and variability in the load which can be included in the model.

In this work we assume:

1. For each equation the error terms ε_j, $j = 1, ..., m$ have a multivariate normal distribution with zero mean and covariance matrix Σ_j that is a positive definite matrix.

2. All elements of X remain fixed (i.e. are not random variables).

Heteroskedasticity refers to a model where the covariance matrix of the error terms are different across equations, that is, in the block diagonal matrix Ω, the non-null matrix are different from each other:

$$Var(\varepsilon) = \Omega = \begin{bmatrix} \Sigma_1 & 0 & ... & 0 \\ 0 & \Sigma_2 & ... & 0 \\ ... & ... & ... & ... \\ 0 & 0 & ... & \Sigma_m \end{bmatrix} = \begin{bmatrix} h^{-1} \times \Lambda_1 & ... & 0 \\ 0 & ... & 0 \\ ... & ... & ... \\ 0 & ... & h^{-1} \times \Lambda_m \end{bmatrix} = \Lambda^{-1} \tag{8}$$

The matrix is written in terms of precision, substituting for Σ_j the terms $h^{-1} \times \Lambda_j^{-1}$ for all $j = 1, ..., m$ where h is the precision (the inverse of the variance $\sigma^2 = h^{-1}$). Moreover, we manage a hierarchical model, since we assume that we do not know the values assumed by the elements of the Ω_i matrices. This model allows to free up the normality assumption, since unknown heteroskedasticity is equivalent to a linear regression model with Student-t errors.

Bayesian technique imposes to set Prior distributions for all parameters of interest, then, the next step will be to choose adequate prior distribution for βs parameters.

3.1 The transformed model

Before discussing the prior and posterior and the computational issues, general results of the model are presented. Since Ω is a positive definite matrix, Cholesky decomposition can be applied, then it exist a $(Nm \times Nm)$ matrix P with the property that $P\Omega P' = I_{Nm}$.

Given the model:

$$y = X\beta + \varepsilon \tag{9}$$

with $\varepsilon \sim N(0, h^{-1} \times \Omega)$

if we multiply both sides of the previous equation by P, we obtain the transformed model

$$y^* = X^*\beta + \varepsilon^* \tag{10}$$

where $y^* = Py$, $X^* = PX$ and $\varepsilon^* = P\varepsilon$. It can be verified that $\varepsilon^* \sim N(0, I_{Nm})$. Hence the transformed model falls again into the standard Normal linear regression model. There are two important implications to be discussed.

Using the properties of the multivariate Normal distribution, the likelihood function of transformed model can be seen to be:

$$\begin{aligned} p(y|\beta, h, \Lambda) &= \frac{h^{\frac{Nm}{2}}}{(2\pi)^{\frac{N\cdot m}{2}}}|\Lambda|^{\frac{1}{2}} \exp\left[-\frac{h}{2}(y - X\beta)'\Lambda(y - X\beta)\right] = \\ & \frac{h^{\frac{Nm}{2}}}{(2\pi)^{\frac{N\cdot m}{2}}} \exp\left[-\frac{h}{2}(y^* - X^*\beta)'(y^* - X^*\beta)\right] \end{aligned} \tag{11}$$

Usually Bayesian technique suggests to use natural conjugate priors, making the β's distribution be dependent Ω, in this way the joint posterior distribution would become: $p(\beta, \Omega) = p(\beta|\Omega)p(\Omega)$. This joint prior has the advantage to derive analytically tractable joint posterior distributions whose main summary statistics are available, sparing in this way the use of posterior simulator: However, the natural conjugate prior for the SUR model has been found by many to be too restrictive. The prior covariances between coefficients in each pair of equations are in fact all proportional to the same matrix. For this reason, following the mainstream literature, here I apply the extended version of the natural conjugate prior: the independent Normal - Wishart prior:

$$p(\beta) = N(\beta, V) \tag{12}$$

$$p(h) = G(\nu_0, s_0^{-2}) \tag{13}$$

Moreover, we assume that β and h have distributions independent on Ω, whose prior will be defined later.

$$p(\beta, h, \Lambda) = p(\beta)p(h)p(\Lambda) \tag{14}$$

Prior hyperparameter elicitation comes from the previous empirical study of Bigerna & Bollino [7]. Then, for the beta parameters I used a Normal Prior distribution centered on the frequentist hourly estimates referring to the previous year (2010).

The joint posterior distribution of all parameters is, as always, the likelihood function times the priors:

$$\begin{aligned} p(\beta, h, \Lambda|y) \;\propto\; & p(\Lambda) \times \\ & \exp\left\{\left[-\frac{h}{2}(y^* - X^*\beta)'(y^* - X^*\beta)\right]\right\} \\ & \times \exp\left\{-\frac{1}{2}(\beta - \beta_0)'V_0^{-1}(\beta - \beta_0)\right\} \\ & \times h^{\frac{1}{2}(Nm+\nu_0-2)} \exp\left[-\frac{h\nu_0}{2s^{-2}}\right] \end{aligned} \tag{15}$$

It is not ascribable to a well-known functional form. However, functional form of the full conditional posterior distributions are known: the full conditional for β is a Normal:

$$\beta|y, h, \Lambda \sim N(\beta_n, V_n) \tag{16}$$

where:

$$V_n = (V_0^{-1} + hX\Lambda^{-1}X)^{-1} \tag{17}$$

and

$$\beta_n = V_n(V_0^{-1}\beta_0 + hX\Lambda^{-1}X\widehat{\beta}(\Lambda)) \tag{18}$$

with

$$\widehat{\beta}(\Lambda) = (X^{*\prime}X^*)^{-1}X^{*\prime}y^* = (X'\Lambda X)^{-1}X'\Lambda y \tag{19}$$

The posterior distribution of h conditional on the other parameters in the model is a Gamma:

$$h|y, \beta, \Lambda \sim G(s_n^{-2}, \nu_n) \tag{20}$$

where:

$$\nu_n = Nm + \nu_0 \tag{21}$$

and

$$s_n^2 = \frac{(y - X\beta)'\Lambda^{-1}(y - X\beta) + \nu_0 s_0^2}{\nu_n} \tag{22}$$

Conditioning on Λ, the two full conditional distributions for β and h combine data and prior information. Given Λ, the full conditional distributions of

β and h are ascribable to a well-known analytical form, as in the traditional linear model, the Gibbs Sampling algorithm can exploit this two densities and constructs a stationary Markov Chain. The first part of simulation procedure has been defined. However, the full conditional posterior of Λ does not take any recognizable form, then the joint posterior distribution $p(\beta, h, \Lambda|y)$ keeps remaining un-tractable since $p(\beta, h, \Lambda|y) \neq p(\beta, h|\Lambda, y) \cdot p(\Lambda|\beta, h, y)$. The inference related to the random parameters β, h and Λ has not been possible yet: the prior distribution for Λ needs to be investigated in order to design the second component of simulation.

When Λ is an unknown parameter, the elements of the matrix Ω in (8) become random variables. The treatment of heteroskedasticity of unknown form is a challenging task and involves the use of a hierarchical prior.

Introducing unknown heteroskedasticity increases the number of parameters to be estimated; if we treat $\Lambda_1, ..., \Lambda_m$ as completely independent and unrestricted matrices, we would not have enough observations to estimate each one of them. For this reason the exchangebility of Λ_i becomes an assumption essential to deal with this high dimensional model. The prior for Λ becomes:

$$p(\Lambda) = \prod_{j=1}^{m} f_W(\Lambda_j|\Lambda_0, \nu_\lambda) \tag{23}$$

which states that Λ_js are different from one another but they are *i.i.d.* draws from the same Wishart distribution (the hierarchical prior). The hierarchical prior imposes a structure to the model that preserves flexibility and makes estimation be possible. The use of a Wishart prior distribution allows to turn out a linear regression model with *i.i.d.* Student-t error terms with $\nu_\lambda > m$ degrees of freedom.

In other words :

$$\begin{aligned} \varepsilon_i|\Lambda_i^{-1} &\sim N(0, \sigma_i^2\Lambda_i^{-1}) && (24)\\ \Lambda_i &\sim W(\Lambda_0, \nu_\lambda) && (25)\\ \varepsilon_i &\sim t(0, \sigma^2, \nu_\lambda) && (26) \end{aligned}$$

The model becomes more flexible since Student-t distribution that is a more general class of distributions that includes Normal density as a special case (occurring when the degrees of freedom ν_λ tent to infinity).

Our treatment of unknown heteroskedasticity is equivalent to a scale mixture of Normal. The error terms ε_i are distributed according to a mixture of m different normal distributions. That is:

$$\varepsilon_i = \sum_j e_{ij} \left(\alpha_r + (H_j)^{-1/2} \eta_{ij} \right) \tag{27}$$

where η_{ij} is i.i.d $N(0, I_m)$ for $i = 1, ..., N$, $j = 1...J$ and e_{ij}, α_j and H_j are all parameters. The e_{ij} is a dichotomous random variable and indicates which distribution component in the mixture the ith error is drawn from.

$$e_{ij} = \begin{cases} 1 & \text{if } \varepsilon_j \sim N(\alpha_j, H_j) \\ 0 & \text{otherwise} \end{cases} \tag{28}$$

Since it is unknown which component the ith error is drawn from, we define $p_j = P(e_{ij} = 1)$ for $j = 1, ..., m$ the probability of the error being drawn from the jth component in the mixture. Formally it means that e_{ij} are i.i.d draws from a Multinomial distribution

$$e_i \sim M(1, p) \tag{29}$$

where $p = (p_1...p_m)'$ is the probability vector p.

The assumption that Λ_i follows a Wishart distribution and that, given Λ_i, the errors are independent Normal $(0, h^{-1}\Lambda_i^{-1})$ is equivalent to the assumption that the distribution of error term ε is a weighted average of Normals having different variances but the same means (i.e. all errors have mean equal to zero). When we mix the error terms' normal distributions using $f_W(\Lambda_i|\Lambda_0, \nu_\lambda)$, they end up to be equal to the Student-t distribution. Intuitively, assuming that a Normal model is too restrictive, a more flexible distribution taking a mixture (the weighted average) of Normals can be created. As more and more Normals are mixed, as the distribution becomes more and more flexible and can approximate any distribution with high degree of freedom. Mixtures of Normal are powerful tool to be used when economic theory does not suggest any particular form of likelihood function and you wish to be more flexible.

However, this model uses a finite mixture of Normal and it cannot be considered non-parametric in the sense that it can not accomodate any distribution, it is "just an extremely flexible modelling strategy" (Koop [17]).

Parameter ν_λ is not known and Bayesian framework imposes to define a prior distribution $p(\nu_\lambda)$. The prior of λ is specified in two steps, firstly we specify $p(\Lambda|\nu_\lambda) = \prod_{i=1}^{N} f_W(\Lambda_i|\Lambda_0, \nu_\lambda)$, secondly we define $p(\nu_\lambda)$; in this way these two steps refer to a hierarchical prior model. $p(\Lambda|\nu_\lambda)$ and $p(\nu_\lambda)$ are the features necessary to design the second part of the simulation procedure.

Let it focus on $p(\Lambda|y, \beta, h, \nu_\lambda)$ and $p(\nu_\lambda|y, \beta, h, \Lambda)$.

$$p(\Lambda|y,\beta,h,\nu_\lambda)=\prod_{i=1}^{N}p(\Lambda_i|y,\beta,h,\nu_\lambda) \tag{30}$$

where

$$p(\Lambda_i|y,\beta,h,\nu_\lambda)=W\left((\nu_\lambda+m)\left[h(\varepsilon_i\varepsilon_i')\right]^{-1}+\nu_\lambda,\nu_\lambda+m\right) \tag{31}$$

Conditional on β, ε_i can be calculated and hence also the parameters of the Gamma density can be sampled within the Gibbs sampler.

Problems arise in the derivation of full conditional posterior for ν_λ. Since ν_λ is positive we assume as a prior an exponential distribution that is a gamma with two degree of freedom:

$$p(\nu_\lambda)=G(\nu_0,2) \tag{32}$$

Then, the full conditional posterior is:

$$p(\nu_\lambda|y,\beta,h,\Lambda)=p(\nu_\lambda|\Lambda)\propto p(\Lambda|\nu_\lambda)p(\nu_\lambda)^1 \tag{33}$$

The kernel of the posterior conditional of ν_λ is simply (30) times (32):

$$\begin{gathered}p(\nu_\lambda|\Lambda)\propto p(\Lambda|\nu_\lambda)p(\nu_\lambda)\\ \propto\left(\frac{\nu_\lambda}{2}\right)^{\frac{N\nu_\lambda}{2}}\Gamma\left(\frac{\nu_\lambda}{2}\right)^{-N}\exp(-\eta\nu_\lambda)\end{gathered} \tag{34}$$

where

$$\eta=\frac{1}{\nu_0}+\frac{1}{2}\sum_{i=1}^{N}\left[\ln|\Lambda|_i^{-1})+tr|\Lambda_0^{-1}\Lambda_i\right] \tag{35}$$

The density derived in (34) is not again a standard one, so algorithm for the posterior simulation of the degree of freedom need to be performed. The simulation strategy is the following:

- Use Random-Walk-Metropolis-Hastings to simulate a sample from $p(\nu_\lambda|\Lambda)$.
- Given ν_λ, run Gibbs Sampling simulating $p(\beta,h,\Lambda,\nu_\lambda|y)$ using the $\{\nu_\lambda^t\}$ sample, $p(\beta|y,h,\Lambda)$ in (16) and $p(h|y,\beta,\Lambda)$ in (20).

[1] Since ν_λ does not eneter in the likelihood $p(\nu_\lambda|y,\beta,h,\lambda)=p(\nu_\lambda|\lambda)$.

The candidate generating function is $q(\nu_{\lambda}^{(s-1)};\nu_{\lambda}^{*}) = N([\nu_{\lambda}^{*} - \nu_{\lambda}^{(s-1)}], 0.2)$ and number of replications are set equal to 11000.

Setting the number of replication equal to 11000 guarantees that the chain takes enough steps to cover all the parameter space, diagnsotic procedure shows satisfactory results: the convergence diagnostic tests performed do not refuse the null hypotesis of convergence to the posterior density. After discarding the first 1000 realizations, the chain $\left\{\beta^i, h^i, \nu_{\lambda}^i\right\}_{i=1001}^{11000}$. The sequence simulate a sample from the posterior $p(\beta, h, \Lambda, \nu_{\lambda}|y)$.

4 Empirical Results

Data refers GME daily data for the 2011 and they had been collected in monthly dataset starting from January 2011 to December 2011. Each monthly dataset accounts for 1.5 millions of raw observations and Bid pertaining the demand side are about 400-450 thousand observations, the 20-25% of the total amount of offers. In each hour of the two years I ranked the bids according to the merit order (price descending order); I included also the rejected bids in order to have the estimation of elasticities relative to the prices of demand curve lower than equilibrium price. These latter elasticities represent in fact the real responsiveness to change in price of purchaser less incline to buy. Then, I aggregated all inelastic bids (bids with submitted price equal to 3000), computing in this way the market point of demand corresponding to the intercept. Finally I derived the remaining downward sloping market demand curve by horizontal sum of bids characterized by the same price.

At the end of the procedure each monthly Dataset accounts for a sample size ranging from 15558 observations in February to 23148 observations in November 2011. For each system of equations we derive the hourly-elasticity for each day of the month by simply adjusting the coefficient related to the log-price regressors through the coefficient related to the daily (iteration) dummy variables. In this way we have derived all beta elasticity for each hour and each day.

In order to have some statistical summaries we aggregated the later estimates in the hourly average elasticity for each month [2]. Firstly, it can be noticed that average elasticities vary within the hours of the day. In 2011 estimates go from a minimum value of from -0.14, recorded in September to -0.0360 recorded in November Moreover, allowing the variance to differ across observations of the same equations has led the peak hour elasticities to be, on average, higher than off-peak ones. The model confirms the Bollino and

[2] For a comparison with estimates derived using an Homoskedastic model see [12]

Bigerna's results. Comparing to the off-peak estimates, peak hour elasticities show higher variability, going from -0.14 to -0.0484, while the off-peak ones vary between -0.070 and -0.0359. Secondly, in 2011 elasticities are higher during the off-peak hours. In the peak hours period, electricity quantities traded are greater than the average as it is shown by the previous tables and confirmed by high frequency of congestion.

[Table 1 - Table 2 here]

As regards to zone segmentation, we computed average elasticities within the group of hours having the same number of zone segmentation after congestion. Estimates shows that during Peak hours higher elasticities has in fact been recorded when the single market occurred. When the transmission constraints are violated, elasticity becomes lower and this is particular evident during the peak hours. High levels of demand causing congestion reduces the responsiveness to change in price. Moreover, frequent congestions during peak hour may suggest that electricity is an essential commodity whose demand is stiff and whose consumption can not be postponed.

Off-peak estimates instead, do not show a well defined behaviour. During off-peak hours electricity is allocated essentially for domestic uses and it means that in this model end-consumers have take the same behaviour with respect to market segmentation and the risk of congestion.

Elasticities aggregated by PUN percentiles are higher when lower levels of price have been recorded. Lower price levels mean low quantities traded and lower income levels. Then, as I said before, consumers with limited expenditure availability (referring essentially to domestic user) have more flexible behaviour given changes in price.

[Table 3 - Table 5 here]

The decomposition of average elasticities between peak and off-peak hours shows that within peak hours, the lower elasticities were recorded when both single market and maximum segmentation market (four zone division) occurred. The lowest average elasticity was recorded in the presence of maximum segmentation of national market that gives evidence of higher levels of demand and income. As we said before, high levels of expenditure availability affect demand elasticity reducing the responsiveness to change in price. Moreover, congestions during the peak hours may suggest that electricity is an essential commodity whose demand is stiff and whose consumption can not be postponed.

[Table 6 - Table 10 here]

5 Conclusions

The models proposed highlights that buyers in the Italian Wholesale Electricity Market react to change in price since the estimated elasticities are different from zero.

Moreover, the estimates differ from one another during the day, on the strenght of the level of electricity loads, the market segmentation structure and the levels of PUN.

In the Heteroskedastic Multivariate Linear Model, the elasticities recorded during peak hours are higher than Off-Peak estimates. Moreover buyers mantain their higher reactivity to changes in price when there is not congestion and when PUN records low values.

Further development of the research may be the application of the heteroskedastic model to the data referring the 2012. Moreover, also the computational part can be implemented. Heteroskedastic model represents a novel in the empirical analyses, but the multi-dimension of the inferential problem made the construction of the algorithm the most challenging task of my thesis. The heteroskedastic model designs for empirical data a statistical framework rigorous and detailed. However, posterior simulation requires the discretional choice of the proposal density and the tuning of its parameters, first of all the variance, affecting the behaviour of the chain and the resulting posterior. Running the procedure with other parameters and alternative functional forms of the candidate generating density could be a possible development in order to make a comparison between the different derived estimates.

6 Tables

	Hour	January	February	March	April	May	June	July	August	September	October	November	December
Peak	9	-0.0756	-0.0699	-0.0869	-0.0676	-0.0853	-0.0495	-0.0506	-0.0730	-0.1388	-0.0579	-0.0603	-0.0599
	10	-0.0764	-0.0696	-0.0864	-0.0679	-0.0855	-0.0492	-0.0509	-0.0731	-0.1321	-0.0582	-0.0598	-0.0600
	11	-0.0771	-0.0695	-0.0867	-0.0682	-0.0859	-0.0484	-0.0510	-0.0769	-0.1387	-0.0582	-0.0599	-0.0579
	12	-0.0771	-0.0705	-0.0872	-0.0682	-0.0863	-0.0485	-0.0504	-0.0728	-0.1342	-0.0578	-0.0600	-0.0556
	13	-0.0769	-0.0707	-0.0877	-0.0677	-0.0868	-0.0490	-0.0505	-0.0754	-0.1355	-0.0568	-0.0599	-0.0571
	14	-0.0761	-0.0701	-0.0876	-0.0686	-0.0863	-0.0493	-0.0503	-0.0777	-0.1416	-0.0569	-0.0603	-0.0579
	15	-0.0759	-0.0701	-0.0878	-0.0677	-0.0859	-0.0498	-0.0505	-0.0758	-0.1374	-0.0571	-0.0607	-0.0615
	16	-0.0758	-0.0706	-0.0875	-0.0682	-0.0852	-0.0496	-0.0508	-0.0756	-0.1398	-0.0578	-0.0602	-0.0588
	17	-0.0751	-0.0707	-0.0872	-0.0678	-0.0853	-0.0488	-0.0512	-0.0757	-0.1371	-0.0579	-0.0605	-0.0610
	18	-0.0751	-0.0706	-0.0868	-0.0676	-0.0859	-0.0486	-0.0514	-0.0767	-0.1369	-0.0578	-0.0611	-0.0612
	19	-0.0759	-0.0703	-0.0869	-0.0679	-0.0856	-0.0493	-0.0520	-0.0795	-0.1387	-0.0586	-0.0610	-0.0613
	20	-0.0753	-0.0710	-0.0869	-0.0688	-0.0856	-0.0499	-0.0516	-0.0782	-0.1434	-0.0582	-0.0611	-0.0581
Off-Peak	21	-0.0571	-0.0528	-0.0452	-0.0580	-0.0630	-0.0438	-0.0594	-0.0577	-0.0498	-0.0707	-0.0395	-0.0607
	22	-0.0572	-0.0541	-0.0456	-0.0592	-0.0657	-0.0434	-0.0588	-0.0576	-0.0487	-0.0706	-0.0419	-0.0612
	23	-0.0573	-0.0556	-0.0489	-0.0580	-0.0629	-0.0432	-0.0592	-0.0576	-0.0495	-0.0676	-0.0447	-0.0596
	24	-0.0576	-0.0522	-0.0459	-0.0613	-0.0662	-0.0427	-0.0588	-0.0577	-0.0498	-0.0664	-0.0418	-0.0595
	1	-0.0576	-0.0553	-0.0455	-0.0595	-0.0650	-0.0432	-0.0587	-0.0578	-0.0496	-0.0701	-0.0422	-0.0601
	2	-0.0581	-0.0557	-0.0470	-0.0603	-0.0679	-0.0439	-0.0588	-0.0578	-0.0503	-0.0675	-0.0460	-0.0599
	3	-0.0580	-0.0559	-0.0504	-0.0619	-0.0632	-0.0438	-0.0588	-0.0577	-0.0514	-0.0645	-0.0372	-0.0600
	4	-0.0579	-0.0556	-0.0521	-0.0641	-0.0642	-0.0439	-0.0590	-0.0577	-0.0506	-0.0668	-0.0365	-0.0604
	5	-0.0573	-0.0532	-0.0526	-0.0594	-0.0665	-0.0429	-0.0593	-0.0569	-0.0504	-0.0622	-0.0359	-0.0607
	6	-0.0567	-0.0519	-0.0517	-0.0584	-0.0635	-0.0429	-0.0591	-0.0569	-0.0510	-0.0675	-0.0362	-0.0603
	7	-0.0570	-0.0531	-0.0500	-0.0582	-0.0666	-0.0435	-0.0593	-0.0569	-0.0509	-0.0689	-0.0360	-0.0609
	8	-0.0572	-0.0526	-0.0475	-0.0624	-0.0676	-0.0441	-0.0591	-0.0565	-0.0513	-0.0675	-0.0366	-0.0611

Tab. 1: Hourly Average Elasticity. 2011.

Average Elasticity	2011			
	I Quarter	II Quarter	III Quarter	IV Quarter
Peak	-0.0778	-0.0677	-0.0882	-0.0591
Off-Peak	-0.0533	-0.0562	-0.0556	-0.0558

Tab. 2: Average Elasticity by Quarter. 2011.

Percentile	January		February		March		April		May		Jun	
	Peak	Off-Peak	Peak	Off-Peak	Peak	Off-Peak	Peak	Off-Peak	Peak	Off-Peak	Peak	Off-Peak
10	-0.0703	.	-0.0710	.	-0.0858	-0.0723	-0.0656	-0.0438	-0.0832	-0.0742	-0.0520	-0.0574
9	-0.0738	.	-0.0694	-0.0374	-0.0861	-0.0502	-0.0695	-0.0664	-0.0878	-0.0587	-0.0518	-0.0439
8	-0.0742	-0.0692	-0.0708	-0.0630	-0.0854	-0.0406	-0.0709	-0.0613	-0.0859	-0.0715	-0.0497	-0.0447
7	-0.0762	-0.0664	-0.0685	-0.0508	-0.0877	-0.0597	-0.0685	-0.0714	-0.0883	-0.0731	-0.0481	-0.0447
6	-0.0796	-0.0560	-0.0700	-0.0512	-0.0862	-0.0594	-0.0646	-0.0587	-0.0859	-0.0682	-0.0485	-0.0473
5	-0.0761	-0.0542	-0.0697	-0.0558	-0.0880	-0.0299	-0.0695	-0.0600	-0.0826	-0.0729	-0.0514	-0.0466
4	-0.0763	-0.0552	-0.0704	-0.0522	-0.0887	-0.0430	-0.0666	-0.0566	-0.0836	-0.0585	-0.0456	-0.0404
3	-0.0718	-0.0575	-0.0729	-0.0565	-0.0902	-0.0499	-0.0670	-0.0607	-0.0842	-0.0633	-0.0496	-0.0455
2	-0.0716	-0.0613	-0.0753	-0.0498	-0.0892	-0.0535	-0.0719	-0.0619	-0.0812	-0.0657	-0.0457	-0.0403
1	.	-0.0547	.	-0.0568	.	-0.0478	-0.0713	-0.0590	-0.0799	-0.0570	-0.0394	-0.0435

Tab. 3: Monthly Peak Average Elsticity by PUN Percentile. Jan.-Jun.2011

Percentile	July		August		September		October		November		December	
	Peak	Off-Peak	Peak	Off-Peak	Peak	Off-Peak	Peak	Off-Peak	Peak	Off-Peak	Peak	Off-Peak
10	-0.0505	-0.0648	-0.0840	-0.0480	-0.1652	-0.0498	-0.0577	-0.0715	-0.0609	-0.0384	-0.0620	-0.0618
9	-0.0531	-0.0685	-0.0840	-0.0612	-0.1211	-0.0501	-0.0570	-0.0614	-0.0608	-0.0449	-0.0600	-0.0620
8	-0.0490	-0.0604	-0.0721	-0.0588	-0.1273	-0.0552	-0.0586	-0.0766	-0.0589	-0.0305	-0.0536	-0.0583
7	-0.0522	-0.0578	-0.0749	-0.0565	-0.1157	-0.0483	-0.0574	-0.0651	-0.0608	-0.0470	-0.0481	-0.0612
6	-0.0513	-0.0613	-0.0732	-0.0533	-0.0808	-0.0443	-0.0571	-0.0696	-0.0588	-0.0412	-0.0630	-0.0566
5	-0.0511	-0.0654	-0.0736	-0.0563	-0.1743	-0.0480	-0.0601	-0.0668	-0.0585	-0.0426	-0.0641	-0.0644
4	-0.0494	-0.0603	-0.0715	-0.0618	-0.1503	-0.0507	-0.0593	-0.0638	-0.0621	-0.0385	-0.0636	-0.0699
3	-0.0516	-0.0581	-0.0777	-0.0556	-0.1614	-0.0511	-0.0564	-0.0562	-0.0617	-0.0517	-0.0633	-0.0663
2	-0.0468	-0.0605	-0.0668	-0.0636	-0.1293	-0.0564	-0.0612	-0.0666	-0.0632	-0.0434	-0.0917	-0.0533
1	-0.0552	-0.0526	-0.0692	-0.0599	.	-0.0541	-0.0630	-0.0759	-0.0640	-0.0299	.	-0.0608

Tab. 4: Monthly Average Elsticity by PUN Percentile. Jul.-Dec.2011

Percentile	Mean		Stand. Dev		Min.		Max.	
	Peak	Off-Peak	Peak	Off-Peak	Peak	Off-Peak	Peak	Off-Peak
1	-0.070	-0.054	0.023	0.028	-0.224	-0.131	-0.010	0.000
2	-0.072	-0.052	0.019	0.027	-0.216	-0.130	-0.010	0.000
3	-0.070	-0.053	0.022	0.025	-0.262	-0.125	-0.012	-0.001
4	-0.068	-0.059	0.021	0.024	-0.217	-0.132	-0.001	-0.002
5	-0.070	-0.055	0.027	0.027	-0.292	-0.126	-0.005	0.000
6	-0.070	-0.054	0.032	0.026	-0.291	-0.127	0.000	0.000
7	-0.073	-0.057	0.037	0.024	-0.252	-0.129	0.000	0.000
8	-0.078	-0.056	0.045	0.024	-0.282	-0.124	-0.001	0.000
9	-0.078	-0.057	0.049	0.024	-0.298	-0.119	-0.001	-0.001
10	-0.084	-0.056	0.060	0.024	-0.309	-0.099	-0.002	0.000

Tab. 5: Average Elsticity by PUN Percentile. 2011

Zone	2011-Jan.-Jun.			2011-Jul.-Dec.		
	Average	Peak	Off-Peak	Average	Peak	Off-Peak
5	0	0	0	0	0	0
4	-0.062	-0.068	-0.056	-0.061	-0.057	-0.065
3	-0.060	-0.067	-0.053	-0.067	-0.076	-0.058
2	-0.063	-0.069	-0.056	-0.065	-0.071	-0.059
1	-0.062	-0.070	-0.054	-0.062	-0.070	-0.055

Tab. 6: Average Elasticity by Zone Segmentation. 2011.

Zone	January		February		March		April		May		June	
	Freq.	Av. Elast.	Freq.	Av. Elast.	Freq.	Av. Elast.	Freq.	Av. Elast.	Freq.	Av. Elast.	Freq.	Av. Elast.
5	0	.	0	.	0	.	0	.	0	.	0	.
4	5	-0.077	11	-0.061	4	-0.091	1	-0.065	16	-0.079	17	-0.035
3	91	-0.077	146	-0.061	125	-0.085	110	-0.067	107	-0.084	133	-0.029
2	269	-0.075	178	-0.060	226	-0.088	215	-0.068	225	-0.088	205	-0.034
1	7	-0.077	1	-0.059	17	-0.090	34	-0.071	24	-0.081	5	-0.043

Tab. 7: Average Peak Elasticity by Zone Segmentation. Jan.-Jun. 2011.

Zone	July		August		September		October		November		December	
	Freq.	Av. Elast.	Freq.	Av. Elast.	Freq.	Av. Elast.	Freq.	Av. Elast.	Freq.	Av. Elast.	Freq.	Av. Elast.
5	0	.	0	.	0	.	0	.	0	.	0	.
4	22	-0.065	0	.	10	-0.047	7	-0.056	1	-0.061	2	-0.054
3	152	-0.060	52	-0.067	221	-0.147	146	-0.060	163	-0.061	137	-0.060
2	196	-0.062	311	-0.060	124	-0.132	202	-0.056	156	-0.060	202	-0.059
1	2	-0.076	9	-0.088	5	-0.074	17	-0.064	40	-0.059	31	-0.057

Tab. 8: Average Peak Elasticity by Zone Segmentation. Jul.-Dec. 2011.

Zone	January		February		March		April		May		June	
	Freq.	Av. Elast.	Freq.	Av. Elast.	Freq.	Av. Elast.	Freq.	Av. Elast.	Freq.	Av. Elast.	Freq.	Av. Elast.
5	0	.	0	.	0	.	0	.	0	.	0	.
4	7	-0.055	10	-0.048	3	-0.070	8	-0.058	15	-0.067	18	-0.040
3	117	-0.057	73	-0.052	105	-0.053	152	-0.060	162	-0.061	123	-0.035
2	121	-0.057	135	-0.057	122	-0.042	120	-0.061	165	-0.070	157	-0.051
1	127	-0.058	118	-0.053	141	-0.050	80	-0.060	30	-0.061	62	-0.043

Tab. 9: Average Off-Peak Elasticity by Zone Segmentation. Jan.-Jun. 2011.

Zone	July		August		September		October		November		December	
	Freq.	Av. Elast.	Freq.	Av. Elast.	Freq.	Av. Elast.	Freq.	Av. Elast.	Freq.	Av. Elast.	Freq.	Av Elast.
5	0	.	0	.	0	.	0	.	0	.	0	.
4	25	-0.057	5	-0.079	7	-0.059	7	-0.067	1	-0.079	4	-0.048
3	193	-0.058	162	-0.057	172	-0.054	93	-0.066	94	-0.057	67	-0.052
2	109	-0.060	128	-0.060	99	-0.047	155	-0.069	127	-0.060	153	-0.057
1	45	-0.062	77	-0.051	82	-0.046	117	-0.067	138	-0.051	148	-0.053

Tab. 10: Average Off-Peak Elasticity by Zone Segmentation. Jul.-Dec. 2011.

References

[1] Aalami, H.A., Parsa Moghaddam, M. and Yousefi, G.R. (2010). Modeling and prioritizing demand response programs in power markets, *Electric Power Systems Research,* **80**, pp 426–435.

[2] Albadi, M.H. and El-Saadany, E.F. (2008). A summary of demand response in electricity markets. *Electric Power Systems Research*, **78,** pp. 1989–1996.

[3] Bask, M. and Widerberg, A. (2009). Market structure and the stability and volatility of electricity prices. *Energy Economics* **31**, pp 278–288.

[4] Bianco, V., Manca, O. and Nardini, S. (2009). Electricity consumption forecasting in Italy using linear regression models. *Energy*, **34**, pp 1413-1421.

[5] Bhattacharyya, S. C. and Timilsina, G. R. (2009). Energy Demand Models for Policy Formulation: A Comparative Study of Energy Demand Models. *World Bank Policy Research Working Paper Series,* 4866.

[6] Bernardo, J, M. *Bayesian theory,* Wiley & Sons, 1994,Chichester.

[7] Bigerna, S. and Bollino, C. A. (2014). Electricity Demand in Wholesale Italian Market. *The Energy Journal*, **35,** pp. 25-46.

[8] Brendt, E. R. and D. O. Wood. (1975) Technology, prices and the derived demand for energy, *The Review of Economics and Statistics*, **57**(3), pp. 259-268.

[9] Chib, S., Greenber, E. (1994). Hierarchical analysis of SUR models with extension to correlated serial errors and time-vrying parameter models. *Journal of Econometrics*, **68**, pp. 339-360.

[10] Chib, S., Greenber, E. (1995). Understanding the Metropolis Algorithm. *The American Statistician,* **49,** pp. 327-335.

[11] Creel, M. and Farell, M. (1996). SUR Estimation of multiple time-series models with heteroskedasticity and serial correlation of unknown form. *Economics Letters*, **53**, pp. 239-245.

[12] D'Errico, M.C., Bollino C.A. (2015). Bayesian Analysis of Demand Elasticity in the Italian Electricity Market. *Sustainability* 2015, **7**, pp. 12127-12148.

[13] Gelman, A., Roberts, G. O., Gilk, W. R., (1994). Efficient Metropolis Jumping Rule. BayesianStatistics, **5**, pp.599-607.

[14] Geweke, J. (1989). Bayesian Inference in Econometric Models using Monte Carlo Integration. *Econometrica*, **57**, pp. 1317-1340.

[15] Hasting, W.K. (1970). Monte Carlo Sampling Methods using Markov chain Monte Carlo and their applications. *Biometrika*, **57,** pp. 97-109.

[16] Kirschen, D. S., Strbac, G., Cumperayot, P. and de Paiva Mendes, D.(2000). Factoring the Elasticity of Demand in ElectricityPrices. IEEE Transaction on Power System, **15** (2), pp 612-617.

[17] Koop, G. (2003). *Bayesian Econometrics*, Wiley.

[18] Koop, G., Poirier, D. J. and Tobas, J. L.(2007). *Bayesian Eocnometric Methods.* Cambridge University Press.

[19] Metropolis, N., Rosebluth, A.W., Rosebluth, M.N., Teller, A.H. and Teller, E. (1953). Equations of State Calculations by Fast Computing Machines. *Journal of Chemical Physics* **21**, pp. 1087-1092.

[20] Patrick, R. H. and Wolak, F. *Estimating the Customer-Level Demand for Electricity Under Real-Time Market Prices*, Preliminary Draft.

[21] Ravikumar, B., Surajit R. and Eugene Savin N.(2000). Robust Wald Tests in SUR System with Adding-up Restriction, *Econometrica*, **68**, pp 715-719.

[22] Taylor, L. D. (1975). The Demand for Electriciy: A Survey. *The Bell Journal of Eonomics,* **6 (**1), pp 74-110.

[23] Tierney, L. (1994). Markov Chain for Exploring Posterior Distributions. *Annals of Statistics*, **22**, pp 1701-1762.

[24] Tomohiro, A. (2011). Bayesian Variable Selection for the Seemingly Unrelated Regression Models with a large numberof predictors. *Japan Statics Society*, **41**, pp 187-203.

[25] Tomohiro, A. , Zellner, A. (2010). Hierarchical Bayesian Analysis of the Seemingly Unrelated Regression and Simulation Equatins Model using a Combination of Direct Montecarlo and Importance Sampling Techniques. *Bayesian Analysis*, **5**, pp. 65-96.

[26] Wang, J., S. Kennedy, and J. Kirtley. "A new wholesale bidding mechanism for enhanced demand response in smart grids. Institute of Electrical and Electronics Engineers Press, 2010.

[27] Willems, B., Rumiantseva I. and Weigt, H. Cournot versus Supply Functions: What does the data tell us?. *Energy Economics*, **31,** pp 38-47.

[28] Wolak F.A and R.H. Patrick (1997). "Estimating the Consumer-Level Demand for Electricity Under Real-Time Market Prices," mimeo, Stanford University

[29] Zellner, A. and Tomohiro, A. (2010). A direct Monte Carlo approach for Bayesian analysis of the seemingly unrelated regression model. *Jounal of Econometrics,* **159**, pp. 33-45